~~#2~~ #4 [1]

Let $\Omega \subset\subset \mathbb{R}^N$ open bounded set with smooth boundary.

Suppose $u \; (= u(x,t))$ solves

$$\begin{cases} u_t = \Delta u & (x,t) \in \Omega \times (0,\infty) \\ \partial_\nu u = 0 & (x,t) \in \partial\Omega \times (0,\infty) \\ u(x,0) = \phi(x) & x \in \Omega \end{cases}$$

Let $m(t) := \int_\Omega u(x,t)\,dx$ denote the "mass".

Show $m'(t) = 0 \quad \forall t > 0$.

You can assume stuff sufficiently nice so that all integration by parts (& the like) are valid.

Question #5

Consider the non-linear wave eq.

$$\begin{cases} u_{tt} - \Delta u = u^3 & (x,t) \in \Omega \times (0,\infty) \\ u = 0 & (x,t) \in \partial\Omega \times (0,\infty) \\ u(x,0) = \phi(x) & x \in \Omega. \\ u_t(x,0) = \psi(x). \end{cases}$$

Suppose u a solution.

Show the following is a "conserved quantity" ie $E'(t) = 0$

$$E(t) = \frac{1}{2}\int_\Omega u_t^2\,dx + \frac{1}{2}\int_\Omega |\nabla u|^2\,dx - \frac{1}{4}\int_\Omega u^4\,dx.$$

Again Assume everything nice enough to justify various computations.

~~Question 3~~