Math 314/814–002: Matrix Theory **Dr. S. Cooper, Fall 2008**

Name: ______________________________

Quiz 13 Solutions

Please write your solutions to the following exercises in the space provided. You should write legibly and fully explain your work.

Good Luck!

(1) Let V be the vector space $V = \{p(x) \in \mathcal{P}_2 : p(1) = 0\}$.

(a) Find a basis for V. Be sure to show all of your work. [8 pts]

Solution: Note that

$$\begin{aligned} V &= \{p(x) \in \mathcal{P}_2 : p(1) = 0\} \\ &= \{p(x) = a + bx + cx^2 \in \mathcal{P}_2 : p(1) = 0\} \\ &= \{p(x) = a + bx + cx^2 \in \mathcal{P}_2 : a + b + c = 0\} \\ &= \{p(x) = a + bx + (-a - b)x^2 \in \mathcal{P}_2\} \\ &= \{p(x) = a(1 - x^2) + b(x - x^2) \in \mathcal{P}_2\} \\ &= span(1 - x^2, x - x^2) \end{aligned}$$

We now verify that our spanning polynomials are also linearly independent. So, suppose we have scalars c_1 and c_2 such that

$$c_1(1 - x^2) + c_2(x - x^2) = 0 + 0x + 0x^2.$$

Equivalently,

$$c_1 + c_2x + (-c_1 - c_2)x^2 = 0 + 0x + 0x^2.$$

Comparing coefficients, we see that $c_1 = c_2 = 0$. Thus,

$$\mathcal{B} = \{1 - x^2, x - x^2\}$$

is a basis for V.

(b) What is the dimension of V? [2 pts]

Solution: Since the above basis for V has two polynomials, $\dim(V) = 2$.

(2) In M_{22}, let $\mathcal{B}$ be the standard basis $\mathcal{B} = \{E_{11}, E_{12}, E_{21}, E_{22}\}$ and let $\mathcal{C}$ be the basis $\mathcal{C} = \{A, B, C, D\}$, where

$$A = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}, \quad D = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}.$$

Find the change of basis matrix $P_{\mathcal{C}\leftarrow\mathcal{B}}$. (Hint: You can find the appropriate coordinate vectors by inspection, but show your work and not just the final matrix.) [10 pts]

Solution: We have the linear combinations:

$$\begin{aligned} E_{11} &= A + 0B + 0C + 0D \\ E_{12} &= -A + B + 0C + 0D \\ E_{21} &= 0A - B + C + 0D \\ E_{22} &= 0A + 0B - C + D \end{aligned}$$

So,

$$P_{\mathcal{C}\leftarrow\mathcal{B}} = \begin{bmatrix} [E_{11}]_{\mathcal{C}} & [E_{12}]_{\mathcal{C}} & [E_{21}]_{\mathcal{C}} & [E_{22}]_{\mathcal{C}} \end{bmatrix} = \begin{bmatrix} 1 & -1 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$