Problem Set 9
Due: Thursday, March 31

Work all of the following problems. Remember, you are encouraged to work together on Problem Sets, but each student must turn in his or her own write-up. Be sure to adhere to the Rules and Expectations outlined in the Course Information Sheet.

1   Traditional Problems

1. (Gallian, Chapter 8 Exercises, #4) Let G and H be groups. Show that $G \oplus H$ is Abelian if and only if G and H are Abelian. State the general case.

2. (Gallian, Chapter 8 Exercises, #7) Let G_1 and G_2 be groups. Prove that $G_1 \oplus G_2$ is isomorphic to $G_2 \oplus G_1$. State the general case.

3. (Gallian, Chapter 8 Exercises, #15) Let G and H be groups. If $G \oplus H$ is cyclic, prove that G and H are cyclic. State the general case. (*Note: You may apply the results of Chapter 8, Exercise #3 from Gallian's text.*)

4. (Gallian, Chapter 8 Exercises, #20) Determine the number of elements of order 15 and the number of cyclic subgroups of order 15 in $\mathbb{Z}_{30} \oplus \mathbb{Z}_{20}$.

5. (Gallian, Chapter 8 Exercises, #44) Is $\mathbb{Z}_{10} \oplus \mathbb{Z}_{12} \oplus \mathbb{Z}_6 \approx \mathbb{Z}_{60} \oplus \mathbb{Z}_6 \oplus \mathbb{Z}_2$?

6. (Gallian, Chapter 8 Exercises, #47) How many isomorphisms are there from $\mathbb{Z}_{12}$ to $\mathbb{Z}_4 \oplus \mathbb{Z}_3$?

7. Subgroups and External Direct Products:

   (a) Let G_1 and G_2 be groups, let H_1 be a subgroup of G_1 and let H_2 be a subgroup of G_2. Prove that $H_1 \oplus H_2$ is a subgroup of $G_1 \oplus G_2$.

   (b) Find a subgroup of $\mathbb{Z}_9 \oplus \mathbb{Z}_3$ that is not of the form $H \oplus K$ for any subgroup H of $\mathbb{Z}_9$ and any subgroup K of $\mathbb{Z}_3$.