# Problem Set 12
# Due: Thursday, April 28

Work all of the following problems. Remember, you are encouraged to work together on Problem Sets, but each student must turn in his or her own write-up. Be sure to adhere to the Rules and Expectations outlined in the Course Information Sheet.

## 1 Traditional Problems

1. (Gallian, Chapter 24 Exercises, #4) Show that $\text{cl}(a) = \{a\}$ if and only if $a \in Z(G)$.

2. (Gallian, Chapter 24 Exercises, #5) If $|G| = 36$ and $G$ is non-Abelian, prove that $G$ has more than one Sylow 2-subgroup or more than one Sylow 3-subgroup.

3. (Gallian, Chapter 24 Exercises, #11) Suppose that $G$ is a group and that $|G| = p^n m$, where $p$ is prime and $p > m$. Prove that a Sylow $p$-subgroup of $G$ must be normal in $G$.

4. (Gallian, Chapter 24 Exercises, #45) If $G$ is a group of odd order and $x \in G$, show that $x^{-1}$ is not in $\text{cl}(x)$.

5. (Gallian, Chapter 24 Exercises, #49) Show that $\mathbb{Z}_2$ is the only group that has exactly two conjugacy classes.

6. Let $G$ be a finite group and suppose that $|G : Z(G)| = n$. Show that $|\text{cl}(x)| \leq n$ for all $x \in G$.

7. (Gallian, Chapter 26 Exercises, #1) Let $S$ be a set of distinct symbols. Show that the relation defined on $W(S)$ in this chapter is an equivalence relation.