Problem Set 1
Due: Thursday, January 27

Work all of the following problems. Remember, you are encouraged to work together on Problem Sets, but each student must turn in his or her own write-up. Be sure to adhere to the Rules and Expectations outlined in the Course Information Sheet.

1 Traditional Problems

1. (Gallian, Chapter 1 Exercises, #1) With pictures and words, describe each symmetry in D_3 (the set of symmetries of an equilateral triangle).

2. (Gallian, Chapter 1 Exercises, #2) Write out a complete Cayley table for D_3.

3. (Gallian, Chapter 1 Exercises, #3) Is D_3 Abelian? Support your answer with either a proof of a specific counter-example.

4. (Gallian, Chapter 2 Exercises, #6) Give an example of a group and group elements a and b with the property that $a^{-1}ba \neq b$, where a^{-1} denotes the inverse of a.

5. (Gallian, Chapter 2 Exercises, #14) Let G be a group with the following property: Whenever a, b, and c belong to G and $ab = ca$, then $b = c$. Prove that G is Abelian.

6. Let G be a group and let $g \in G$. Define a function $\phi_g : G \to G$ by $\phi_g(x) = gxg^{-1}$, where g^{-1} is the inverse of g, for all $x \in G$. Show that ϕ_g is one-to-one and onto. (Recall that a function f is *one-to-one* if whenever $f(a) = f(b)$ we must have $a = b$. Recall that a function $f : S \to T$ is *onto* if for each $t \in T$ there is an element $s \in S$ such that $f(s) = t$.)