Problem Set 0
Due: Tuesday, January 18

Work all of the following problems. Remember, you are encouraged to work together on Problem Sets, but each student must turn in his or her own write-up. Be sure to adhere to the Rules and Expectations outlined in the Course Information Sheet.

1 Traditional Problems

1. (Gallian, Chapter 0 Exercises, #16) Determine 7^{1000} mod 6 and 6^{1001} mod 7.
2. Use complete mathematical induction to prove that $2^{n-1} \leq n!$ for every non-negative integer n. [Recall that $0! = 1$ and for $n > 0$, $n! = 1 \cdot 2 \cdot 3 \cdots (n-1) \cdot n$.] *Hint:* You will want to start by verifying two base cases: $n = 0$ and $n = 1$.
3. Let $S = \mathbb{R}^2$ be the set of ordered pairs of real numbers and, for (a, b), $(c, d) \in S$, define $(a, b) \sim (c, d)$ if $5a + 9b = 5c + 9d$. Show that $\sim$ is an equivalence relation on S. There is a nice geometric description of the equivalence classes. What is it?

2 Computer Problems

Start by reading through Chapter -1 and Chapter 0 (but stop at the bottom of page 10) of the lab manual. You do not need to turn anything in from these chapters, but you'll probably want to do all the problems just to get a feel for how to use GAP. Then do (to turn in) the following problem:

1. As mentioned in the lab manual, there is a nice combinatorial formula for the sum of the first n integers:
$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2}.$$
There is a similarly nice formula for the sum of the first n cubes. The goal of this problem is to use GAP to conjecture the formula, which you will then prove using induction.
   (a) Define in GAP a function that, given a positive integer n, outputs the sum of the first n cubes. For example, if your function is called f, then $f(1) = 1^3 = 1$, $f(2) = 1^3 + 2^3 = 9$ and $f(3) = 1^3 + 2^3 + 3^3 = 36$. *Hint:* You'll want to use the GAP command "Sum". You can find out how to use this command by typing `?Sum` at a GAP prompt.
   (b) Compute, using your function, the sum of the first n cubes for at least 4 different values of $n \geq 4$. Then use this output to conjecture a nice formula for the sum of the first n cubes.
   (c) Test your conjecture on a value of n that is larger than any of the examples you previously computed.
   (d) Use induction to prove that your conjecture holds for all integers $n \geq 1$.

Some Comments:

- Please intersperse your GAP commands and output with your explanations. You'll certainly want to create a log file as described in Chapter -1. If you type up your solutions, you can cut and paste from this log file into your solution file; please use a different font so it is easy to tell what is what. If you hand-write your solutions, you should still print out your log file; then physically cut and paste it into your solutions.

- In general, be sure to read the questions in the lab manual carefully. Many contain multiple parts that aren't separated out as such.