# Group Projects

**Overview:** Each student is required to participate in a group project. Each group will choose a different project topic. You can pick your own group members, but there should be 4 groups of 3 members each and 2 groups with 2 members each for a total of 6 groups. Each project will have both a written and an oral component. These projects can be based on interesting problems or applications that were considered in class, but which were not fully explored, or they can be chosen from the list of suggested topics below. All participants in a group will receive the same grade, so it is important that each person in the group participate fully and equally. Your project will count 20% of your final grade.

**Written Component:** Each group will hand in 1 paper. Your paper should be about 5 to 10 pages long. Quality is more important than quantity. Have something to say and say it clearly and concisely. If you are presenting the results of your investigation of some journal article or textbook chapter, you should fill in the missing parts of each argument or proof and do any problems left to the reader. It would be better to go into a small part of some topic in depth and detail, rather than try to cover a large area superficially. This is your opportunity to show that you can read some mathematics on your own and then explain it in writing to your reader. *Please do not copy any references - appropriately cite these.*

**Oral Presentations:** You will have 30 minutes to enlighten your colleagues about the topic you have researched. Your presentation should be clear and to the point. Choose your examples carefully to illustrate the points you want to make. In a group presentation, all members in the group should have a role and all should be able to answer any questions which arise. You should rehearse your presentation in advance on some fellow students and leave some time for questions and interruptions. Class presentations always take more time than you think they will. Rehearsal will help you to better gauge how much you can accomplish. *Attendance is mandatory for everyone for all class presentations.* Each absence for these class sessions will reduce your course grade by 1/3 letter.

**Important Dates:** We will adhere to the following deadlines. There will be no extensions.

- By Thursday, October 15 you need to inform me of your group members.
- By Thursday, October 29 you need to inform me of the topic your group will research.
- On Thursday, October 29 we will discuss the group presentation schedule.
- On Tuesday, November 3 the final group presentation schedule will be announced.
- On Thursday, November 19 you will submit a preliminary version of your written paper. I will provide feedback on this for your final submission.
- There will be 2 presentations each class for the last 3 class meetings (Dec. 3, 8 and 10).
- The final group paper is due the same day of the group presentation.

**Resources:** You may want to browse through some of the books on Number Theory in the math library. The internet has many good sites dealing with number theoretic topics. Using any of the standard search engines should produce good references. (Be careful about using websites as references – some are more reliable than others! If you want to use a website as one of your references, you should clear it with me first.) I'm happy to help you find some references if you like.

**Some Possible Topics:**

**Check Digit Schemes:** What schemes are used for some specific types of numbers (credit cards, drivers' licenses, passport numbers, shipping labels)? What are the advantages and disadvantages of certain schemes? Why do you think each of these particular schemes was chosen?

**Mathematics and Magic:** What are some of the many magicians' tricks, especially with cards, that are built upon number theoretic facts?

**Mathematics and Juggling:** How do jugglers describe juggling patterns they use, and what new tricks have been introduced into the juggling repertoire as a result of mathematical analysis of juggling?

**The Gregorian Calendar:** How does it work? How long is a complete period before the sequence of days and dates repeats? How can you find the day of the week for any date?

**Mathematics and Music:** What does Number Theory have to do with piano tuning and the "Well-tempered Klavier"? What was Bach's contribution? What number theoretic problems arise in tuning a piano so that it can be played in any key?

**Random Number Generation:** How does the built-in random number generator in your calculator or your favorite computer language or spreadsheet actually work? What are its strengths and shortcomings? How could it be improved?

**Perfect Numbers and Mersenne Primes:** What are these things? What is the history of perfect numbers? What precisely is the connection between perfect numbers and Mersenne primes? How are Mersenne primes found? What is known about the possibility of odd perfect numbers?

**Factoring Methods:** We have used only trial division to factor numbers. What are some of the other techniques available? The Fermat method and the Monte-Carlo (or Pollard rho) method are both accessible to someone with the background of this course.

**Primality Tests:** What are some of the tests that are used to determine whether a given integer is prime or composite?

**Non-Decimal Number Systems:** How are number systems constructed on bases other than 10? What are the advantages of bases other than 10? Where are other bases in common use?

**Chinese Remainder Theorem:** What does this say, why is it true, and how is it used? How can it be used to perform arithmetic with large numbers?

**Egyptian Fractions:** Our method of writing fractions as ratios of two integers, e.g. $\frac{5}{6}$, is due to Babylonians. The Egyptians used only fractions with numerator 1 and hence would represent $\frac{5}{6}$ as a sum of two fractions $\frac{1}{2} + \frac{1}{3}$. Can all ordinary fractions be represented in the Egyptian fractions? There are many interesting questions here, both arithmetical and historical.

**Continued Fractions:** Consider these strange representations of fractions:

$$\frac{3}{17} = \frac{1}{5 + \frac{1}{1+\frac{1}{2}}} \quad \text{and} \quad \frac{24}{31} = \frac{1}{1 + \frac{1}{3+\frac{1}{2+\frac{1}{3}}}}$$

In fact, every rational number can be written in this form. What about irrational numbers? Why is expressing things in this way useful?

**Geometric Numbers:** We all know the square numbers. What are triangular numbers and pentagonal numbers? What are some of their interesting properties?

**Fibonacci Numbers:** These are a source of many interesting patterns and even have a journal devoted to them. What are they and why are they important? The first few are 1, 1, 2, 3, 5, 8, 13, etc.

***p*-adic Integers:** What are $p$-adic integers? What are some of their properties? What can they be used for?