Using Math to Understand Our World
Project 3
Containing Infectious Diseases

Note: Hand in answers to questions 1-5. For what else to hand in, see the "What to Hand In" sheet.

Have you heard of the *Center for Disease Control*, or *CDC* for short? One of the main missions of the CDC is to prevent and control infectious diseases. In fact, they were first founded as an agency to help control malaria. The CDC does research, makes recommendations to the government, and informs the public about a plethora of infectious diseases: influenza, SARS, SIDS, West Nile virus, lyme disease, and so on.

A few years ago, with the increase of terrorism, the CDC was asked to make recommendations as to how the government should prepare for the possibility of a smallpox attack. If such an attack occurs, what is the best way to contain it, to keep it from turning into an epidemic? The governments main weapons to fight an attack would be vaccination and quarantine. As you may know, the vaccine is not all that safe and people can die from it. So, since an attack is not all that likely, it's not sensible to just go and vaccinate everyone in the country. Aside from health-care workers and military personnel, people will not be vaccinated unless an attack occurs. Another way to stop the spread of the disease is to quarantine everyone who gets it. That would be good, but it is not really possible to quarantine *everyone*. How will officials know who has the disease? Some people will resist quarantine. The government needs to know what vaccination and quarantine rates will keep the outbreak from turning into an epidemic. They need to know how many doses of the vaccine should be stockpiled so that it is immediately available in the event of an attack.

Our goal in this project is to read the CDC report about smallpox and understand at least some of it. To that end, we will start with a simpler disease, *schoolpox* (which Wendy Hines made up). Once we have developed tools to analyze the simpler schoolpox, we will be in a better position to understand the CDCs analysis of smallpox.

Similar mathematics would be used to analyze bird flu. With bird flu, it's not clear that a vaccine can be developed in time, and even if it can be, it's not clear that enough of it can be produced. So decisions will need to be made about how many people to vaccinate, who to vaccinate, how many people to quarantine, and so on. If the right decisions are made, the disease will likely not turn into an epidemic.

Part I: Schoolpox

In this section, we will use some of the techniques used by the CDC to analyze a *schoolpox* outbreak. We will study the effects of various quarantine and vaccination rates on the spread of the disease and come up with a plan to contain an outbreak of schoolpox (i.e., prevent it from turning into an epidemic).

To start, we need to know some basic information about schoolpox. Schoolpox has a 3-day incubation period. The *incubation period* is the first period of the disease. During the incubation period, the virus is in the person, but the person has yet to experience symptoms. People can be contagious during the incubation period (as they are in the common cold), but in schoolpox, as well as in smallpox, people are not contagious during the incubation period. The incubation period is followed by a 5-day *symptomatic period*. In schoolpox, as well as smallpox, symptoms include fever and sores. In schoolpox, the person is contagious during the 2nd and 3rd days of the symptomatic period and each sick person infects on average 3 other people during the course of their illness (the number of people infected, on average, by a single sick individual, is called the *transmission rate*, so in this case the transmission rate is 3). The probability of passing on the disease is equal on the 2nd and 3rd day of the symptomatic period. After the symptomatic period, the infected person goes through a 2-day *recovery period* during which they are not contagious.

Suppose that there is an outbreak of schoolpox at your school. On Monday, 16 students arrive at school in Day 1 of the incubation stage. We will analyze various intervention strategies and see what works best.

(1) Suppose that every day you are able to quarantine 50% of those people who are symptomatic and un-quarantined (here we will assume that "quarantined" is equivalent to staying home). So, for example, on the first day of the symptomatic period only 8 of the sick students come to school and on the second day only 4 of those come (but that day they will infect some other students). With a 50% quarantine rate, will you be able to stop the spread of schoolpox? What will happen? Will the disease spread throughout the entire school or will it be contained? If it will be contained, how long will it take until there are no more sick kids at school? Read the bullets below before you get started.

- We have to decide what to do about rounding. Suppose for example, there are 3 infectious students in attendance on a given day. Each will infect on average 1.5 people that day (why?), so, on average, 4.5 students will be infected. Should we count this as 4 or 5 students? For this model, we will round up when counting newly infected students and when counting the number of students who attend on their first symptomatic day, and we will round down when counting the number of students who attend on their second or subsequent symptomatic day. For example, if we get that 7.8 kids in their first symptomatic day will attend school and 4.8 kids in their second symptomatic day will attend school, we would record these numbers as 8 and 4, respectively. This may sound complicated, but it actually makes the analysis easier (trust me!).

- Note that to determine the course of the disease as it spreads, you must keep track of how many students are in each of 6 different stages of illness progression. What are the 6 stages?
- Assume that students are chosen at random for quarantine, so a student who has had symptoms for only one day is just as likely to stay home as a student who has had symptoms for several days (though once quarantined, a student will remain quarantined until s/he is not symptomatic). This may or may not seem reasonable to you, but remember that in the back of our heads we are thinking of a scenario where thousands of people are ill and are being quarantined by the government, who are probably not keeping track of who has been sick for how long. If this still doesn't seem reasonable then you can implement the quarantine rate in whatever way does seem reasonable to you, as long as you justify your method.
- Assume that students who are newly infected enter Day 1 of the incubation period the day after they are exposed to contagious students.

Work carefully; it's easy to make mistakes.

(2) What if we are only able to quarantine 25%? Will quarantine be enough to stop the outbreak? What will happen? Use the same rounding and quarantine rules as in part (1).

(3) Suppose that data from previous outbreaks of schoolpox show that vaccinating 20% of students decreases the transmission rate from 3 to 2.4. Can we stop the spread of the disease if we vaccinate 20% of the students and quarantine 25%? Again, work carefully and follow the rounding rules religiously. (Assume that we vaccinate on Day 1 - this may seem silly since none of the students have come down with any symptoms yet and you may ask yourself how we would know that schoolpox has hit, but remember we are thinking about bioterrorist attacks in which case we may well know of the attack before anyone actually comes down with symptoms. Also assume that the vaccination is effective 24 hours after it is administered.)

(4) To do these computations, we need to know the transmission rate. But health and government officials need to know the vaccination rate, so that they know how many people to vaccinate. How do we know what transmission rate a certain vaccination rate will give and vice versa? Well, we keep track of what happened in the past. Whenever there is an outbreak of an infectious disease, health officials keep careful track of the number of infected people, the number of vaccines administered, and other important data. Below is a table of data of schoolpox transmission rates for a few vaccination rates. These were obtained by tracking the spread of schoolpox in three different school outbreaks in previous years. In one school, 20% of students were vaccinated as soon as authorities realized that schoolpox had struck. In another school 30% were vaccinated, and in another school 80% were vaccinated. This is all the data we have.

Vaccination Rate	Transmission Rate
20%	2.4
30%	2.1
80%	0.6

How would we figure out the transmission rates for other vaccination rates? In the case of smallpox, authorities would like to use the smallest vaccination rate reasonable - that is, a vaccination rate that stops the spread of the disease in a reasonable amount of time, while minimizing the number of people who have to be vaccinated. This means that researchers would like to try out several different vaccination rates in the model and see what happens (they'd use a computer to do this). For that purpose it would be really handy to have a formula relating transmission rates and vaccination rates that worked for all vaccination rates.

Can you use the data above to devise such a formula? (*Remark: this is your only exercise in this question!*)

(5) Using your formula from part (4), what do you think a good vaccination rate might be for schoolpox, supposing that realistically we can only expect to quarantine 25% of students? You can't really answer this without using a computer program to try out a lot of different vaccination rates, but if you come up with a vaccination rate that works (and is less than 100%), I'll be happy. Using your vaccination rate, if your school has 500 students, how many vaccines should you stockpile in order to be prepared for an outbreak?

Remark: After working through this problem several times, with several different rates, you can probably appreciate the benefits of having a computer program to do these computations. A computer programmer could write such a program pretty easily. Then researchers could use the program to input vaccination and quarantine rates and track the course of the disease. This would help them immensely in making decisions about best strategies. Your formula for transmission and vaccination rates could be easily programmed into the computer so that the computer could compute the transmission rate associated with the vaccination rate inputted by the researcher. In the smallpox study, the researchers used a computer program like this.

(6) If you assumed that the data I gave in the table above is linear, then it is exactly linear, but real data is never that clean (why not?). In real life you might get a table like this:

Vaccination Rate	Transmission Rate
20%	2.51
30%	2.1
80%	0.54

> If you know that transmission rates drop linearly as vaccination rates increase, then you would have to find the line that "fits" the data best. Do this. You'll have to decide (and explain) what your criteria will be for determining whether one line fits the data better than another.

You will probably have many questions as you work through this project. Please post them on your group's Project 3 discussion board.

Part II: Smallpox

After September 11, concern about bioterrorism rose. One of the most likely bioterrorist threats is smallpox. Smallpox is much more threatening than, say, anthrax because it is so contagious. The most obvious way to protect the citizenry of the United States against a smallpox attack is to have everyone vaccinated against smallpox. However, the smallpox vaccine is dangerous and can even cause death. Given that a bioterrorist attack is not really all that likely, the vaccine may not be the way to go. Instead, the government's goal is to stop the spread of smallpox once an attack has occurred. In particular, the government wants to avoid an epidemic.

There are two main ways to stop the spread of a disease: quarantining those who are ill and vaccinating those who are not ill (vaccines usually give immunity within 24 hours). In an ideal world everyone who gets smallpox from the initial attack would be quarantined and then they would not be able to spread the disease at all, but this is impractical. It takes time to identify who is sick and who is not and then to get the sick into quarantine. During that time, sick people may infect others. Also quarantining requires complete cooperation of the affected people. It may be sensible to also vaccinate some people in order to slow down the rate at which the disease spreads. The right combination of quarantine and vaccination can keep the disease from turning into an epidemic and at the same time minimize the number of people who might die from the smallpox vaccine itself.

The U.S. government charged the *CDC* (*Center for Disease Control*) with the task of studying smallpox and coming up with recommendations as to how the government should prepare for an attack and, in the unlikely event of an actual attack, how they should respond.

The CDC began by collecting data about smallpox transmission rates. The *transmission rate* is the number of people a single sick individual will likely infect during the course of his illness. Such information can be obtained by carefully tracking previous outbreaks of the illness. In this case, the CDC used data from smallpox outbreaks in India and Europe in the 1960's and 1970's (the last known case of smallpox occurred in India in 1977). The CDC must also determine during which phases of the illness a person is infectious. One good thing about smallpox is that people are not infectious before the earliest symptoms appear and while the victim may not immediately recognize the early symptoms, a trained professional will. The CDC must also know how effective the vaccination is. Is a vaccinated person 100% protected? 95% protected? How quickly after the vaccine is administered does the person achieve full immunity? What are the odds that a person will die from the vaccine? What are the odds that a person will be infected from an infected person?

The CDC commissioned 4 people: Martin Meltzer, Inger Damon, James LeDuc and Donald Miller, to gather and interpret this data and make recommendations. These four researchers wrote a report about their study and it is online at

`http://www.cdc.gov/ncidod/eid/vol7no6/meltzer.htm`

In their study, they used computers to simulate the spread of the disease given different quarantine and vaccination rates. Their results included the following recommendations: the vaccine should not be distributed widely before an attack, the government should stockpile 40 million vaccines for use in the event of an attack, and the government should plan to quarantine at least 25% of those who come down with the disease.

Details about the governments quarantine and vaccination plans are classified.

Read this report. A reading guide is included in this packet. Answering the reading guide questions will help you get through the report, so read it with the reading guide in one hand. Also, as you will see on the "What to Hand In" sheet, some of your answers to the reading guide questions must be handed in.

Part III: Cholera

Enclosed in this packet is an article entitled *Unfeeling Statistics: Snow on Cholera* by T. W. Korner, a mathematician at Cambridge University. This article is about the efforts of Dr. John Snow in the 1850's to figure out the cause of cholera. In those days, it was not common for people to analyze data as a way of proving or disproving hypotheses, rather hypotheses were usually "proven" or "disproven" via philosophical discussion. Read this article and think about how Snow used good mathematical habits of mind to save a lot of people.