Math 314/814: Matrix Theory **Dr. S. Cooper, Fall 2008**

Homework Solutions – Week of August 26

Section 1.1:

(3) A complete solution to this exercise can be found at the back of your text (page 671).

(5) All of the drawings for this exercise can be found at the back of your text (pages 671 - 672). We compute each of the vectors as follows:

(a) $\overrightarrow{AB} = [4-1, 2-(-1)] = [3,3]$

(b) $\overrightarrow{AB} = [2-0, -1-(-2)] = [2,1]$

(c) $\overrightarrow{AB} = \left[\frac{1}{2}-2, 3-\frac{3}{2}\right] = \left[-\frac{3}{2}, \frac{3}{2}\right]$

(d) $\overrightarrow{AB} = \left[\frac{1}{6}-\frac{1}{3}, \frac{1}{2}-\frac{1}{3}\right] = \left[-\frac{1}{6}, \frac{1}{6}\right]$

(11)

$$\begin{aligned} 2\mathbf{a}+3\mathbf{c} &= 2[0,2,0]+3[1,-2,1] \\ &= [0,4,0]+[3,-6,3] \\ &= [0+3, 4-6, 3+0] \\ &= [3,-2,3] \end{aligned}$$

(12)

$$\begin{aligned} 2\mathbf{c}-3\mathbf{b}-\mathbf{d} &= 2[1,-2,1]-3[3,2,1]-[-1,-1,-2] \\ &= [2,-4,2]+[-9,-6,-3]+[1,1,2] \\ &= [2-9+1, -4-6+1, 2-3+2] \\ &= [-6,-9,1] \end{aligned}$$

(15)

$$\begin{aligned}
2(\mathbf{a}-3\mathbf{b})+3(2\mathbf{b}+\mathbf{a}) &= (2\mathbf{a}-6\mathbf{b})+(6\mathbf{b}+3\mathbf{a}) \ \ \text{(Theorem 1.1, part e)}\\
&= ((2\mathbf{a}-6\mathbf{b})+6\mathbf{b})+3\mathbf{a} \ \ \text{(Theorem 1.1, part b)}\\
&= (2\mathbf{a}+(-6\mathbf{b}+6\mathbf{b}))+3\mathbf{a} \ \ \text{(Theorem 1.1, part b)}\\
&= (2\mathbf{a}+(-6+6)\mathbf{b})+3\mathbf{a} \ \ \text{(Theorem 1.1, part f)}\\
&= (0\mathbf{b}+2\mathbf{a})+3\mathbf{a} \ \ \text{(Theorem 1.1, part a)}\\
&= 0\mathbf{b}+(2\mathbf{a}+3\mathbf{a}) \ \ \text{(Theorem 1.1, part b)}\\
&= (2+3)\mathbf{a} \ \ \text{(Theorem 1.1, part f)}\\
&= 5\mathbf{a}
\end{aligned}$$

(17)

$$\mathbf{x}-\mathbf{a}=2(\mathbf{x}-2\mathbf{a}) \implies \mathbf{x}-\mathbf{a}=2\mathbf{x}-4\mathbf{a} \implies \mathbf{x}=3\mathbf{a}$$

(18)

$$\begin{aligned}
\mathbf{x}+2\mathbf{a}-\mathbf{b} &= 3(\mathbf{x}+\mathbf{a})-2(2\mathbf{a}-\mathbf{b})\\
\implies \mathbf{x}+2\mathbf{a}-\mathbf{b} &= 3\mathbf{x}+3\mathbf{a}-4\mathbf{a}+2\mathbf{b}\\
\implies \mathbf{x}+2\mathbf{a}-\mathbf{b} &= 3\mathbf{x}-\mathbf{a}+2\mathbf{b}\\
\implies -2\mathbf{x} &= -3\mathbf{a}+3\mathbf{b}\\
\implies \mathbf{x} &= \frac{3}{2}\mathbf{a}-\frac{3}{2}\mathbf{b}=\frac{3}{2}(\mathbf{a}-\mathbf{b})
\end{aligned}$$

(21) A complete solution to this exercise can be found at the back of your text (page 672).

Section 1.3:

(3) A complete solution to this exercise can be found at the back of your text (page 673).

(6) (a) The vector equation $\mathbf{x} = \mathbf{p} + t\mathbf{d}$ is

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ 0 \\ -2 \end{bmatrix} + t \begin{bmatrix} 0 \\ 2 \\ 5 \end{bmatrix}.$$

(b) The parametric form is

$$\begin{aligned} x &= 3 \\ y &= 2t \\ z &= -2 + 5t. \end{aligned}$$

(9) A complete solution to this exercise can be found at the back of your text (page 673).

(10) (a) The vector equation $\mathbf{x} = \mathbf{p} + s\mathbf{u} + t\mathbf{v}$ is

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ -4 \\ -3 \end{bmatrix} + s \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} + t \begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix}.$$

(b) The parametric form is

$$\begin{aligned} x &= 6 - t \\ y &= -4 + s + t \\ z &= -3 + s + t. \end{aligned}$$

Section 2.1:

(1) The equation $x - \pi y + \sqrt[3]{5}z = 0$ is linear.

(3) The equation $x^{-1} + 7y + z = \sin\left(\frac{\pi}{9}\right)$ is not linear because it involves the reciprocal x^{-1}.

(5) The equation $3\cos x - 4y + z = \sqrt{3}$ is not linear because it involves the function $\cos x$.

(7) $2x + y = 7 - 3y \iff 2x + 4y = 7$. The linear equation $2x + 4y = 7$ has the same solution set as the given equation.

(9) If $x \neq 0$ and $y \neq 0$, then

$$\begin{aligned} \frac{1}{x} + \frac{1}{y} &= \frac{4}{xy} \\ \iff \frac{y+x}{xy} &= \frac{4}{xy} \\ \iff x + y &= 4. \end{aligned}$$

So, the linear equation $x + y = 4$ where $x, y \neq 0$ has the same solution set as the given equation.

(11)

$$3x - 6y = 0 \iff 3x = 6y \iff x = 2y.$$

Thus, for any real number t, we see that $[x, y] = [2t, t]$ is a solution. Observe that the complete set of solutions corresponds to the set of points on the line determined by the given equation. Setting $y = t$, a parametric solution is given by

$$\begin{aligned} x &= 2t \\ y &= t. \end{aligned}$$

So, the solution set to the equation $3x - 6y = 0$ is

$$\left\{ \begin{bmatrix} 2t \\ t \end{bmatrix} : t \in \mathbb{R} \right\}.$$

(13) The complete set of solutions corresponds to the set of points in the plane determined by the equation $x + 2y + 3z = 4$. Setting $y = s$ and $z = t$, a parametric solution is given by

$$\begin{aligned} x &= 4 - 2s - 3t \\ y &= s \\ z &= t. \end{aligned}$$

So, the solution set to the equation $x + 2y + 3z = 4$ is

$$\left\{ \begin{bmatrix} 4 - 2s - 3t \\ s \\ t \end{bmatrix} : s, t \in \mathbb{R} \right\}.$$

(14) The complete set of solutions corresponds to the set of points in the plane determined by the equation $4x_1 + 3x_2 + 2x_3 = 1$. Setting $x_2 = s$ and $x_3 = t$, a parametric solution is given by

$$\begin{aligned} x_1 &= \frac{1}{4} - \frac{3}{4}s - \frac{1}{2}t \\ x_2 &= s \\ x_3 &= t. \end{aligned}$$

Thus, the solution set to the equation $4x_1 + 3x_2 + 2x_3 = 1$ is

$$\left\{ \begin{bmatrix} \frac{1}{4} - \frac{3}{4}s - \frac{1}{2}t \\ s \\ t \end{bmatrix} : s, t, \in \mathbb{R} \right\}.$$

(15) The geometric solution to this exercise can be found at the back of your text (page 674). Algebraically, we first subtract 2 times the first equation from the second:

$$\begin{aligned} x + y &= 0 \\ -y &= 3. \end{aligned}$$

This gives $y = -3$. Using back substitution, we see that $x - 3 = 0 \implies x = 3$. Thus, given system has the unique solution

$$[3, -3].$$

(16) The graphs of the lines $x - 2y = 7$ and $3x + y = 7$ show that there is a unique solution (i.e. there is one and only one point of intersection between these two lines). Algebraically, we subtract 3 times the first equation from the second:

$$\begin{aligned} x - 2y &= 7 \\ 7y &= -14. \end{aligned}$$

So, $y = -2$. Back substitution then gives that

$$x - 2y = 7 \implies x = 2y + 7 = 2(-2) + 7 = 3.$$

We conclude that the given system has the unique solution

$$[3, -2].$$

(20) We first manipulate the second equation:

$$2v = 6 \implies v = 3.$$

Substituting this into the first equation we have

$$2u - 3v = 5 \implies 2u - 3(3) = 5 \implies 2u = 14 \implies u = 7.$$

Thus, the solution to the given system is

$$[7, 3].$$

(23) We start with the fourth equation which says that $x_4 = 1$. Then we substitute this data into the third equation:

$$x_3 - x_4 = 0 \implies x_3 - 1 = 0 \implies x_3 = 1.$$

Working with the second equation, we now have

$$x_2 + x_3 + x_4 = 0 \implies x_2 = -x_3 - x_4 = -1 - 1 = -2.$$

Finally, we substitute this information together in the first equation:

$$x_1 + x_2 - x_3 - x_4 = 1 \implies x_1 = -x_2 + x_3 + x_4 + 1 = 2 + 1 + 1 + 1.$$

Hence, the solution to the given system is

$$[5, -2, 1, 1].$$

(28) The augmented matrix of the given linear system is

$$\left[\begin{array}{ccc|c} 2 & 3 & -1 & 1 \\ 1 & 0 & 1 & 0 \\ -1 & 2 & -2 & 0 \end{array}\right].$$

(29) A complete solution to this exercise can be found at the back of your text (page 674).

(35) We solve the system using the technique of Example 2.6 from the text. The augmented matrix is:

$$\begin{bmatrix} 1 & 5 & \vert & -1 \\ 0 & 6 & \vert & -6 \\ 1 & 2 & \vert & 2 \end{bmatrix}.$$

We subtract row one from the third row:

$$\begin{bmatrix} 1 & 5 & \vert & -1 \\ 0 & 1 & \vert & -1 \\ 0 & -3 & \vert & 3 \end{bmatrix}.$$

We now divide row three by -3:

$$\begin{bmatrix} 1 & 5 & \vert & -1 \\ 0 & 1 & \vert & -1 \\ 0 & 1 & \vert & -1 \end{bmatrix}.$$

Finally, we subtract row two from row three:

$$\begin{bmatrix} 1 & 5 & \vert & -1 \\ 0 & 1 & \vert & -1 \\ 0 & 0 & \vert & 0 \end{bmatrix}.$$

This gives us the system

$$\begin{aligned} x + 5y &= -1 \\ y &= -1 \\ 0x + 0y &= 0 \end{aligned}$$

which has the same solution set as the given system. Using back substitution we see that $y = -1$ and so $x = 4$. So, the solution to the given system is

$$[4, -1].$$