

Rounding Issues – Clarification

Math 314–002

Applied Mini-Project #1

Some explanation may be helpful for #2 (a) on page 86. Obtaining the “solution” $x = -1.00, y = 1.01$ depends on how one rounds to three significant digits while carrying out the involved calculations.

To solve the system

$$\begin{aligned} 0.400x + 99.6y &= 100 \\ 75.3x - 45.3y &= 30.0 \end{aligned}$$

we row-reduce the associated augmented matrix:

$$\left[\begin{array}{cc|c} 0.400 & 99.6 & 100 \\ 75.3 & -45.3 & 30.0 \end{array} \right] \xrightarrow{R_2 \rightarrow R_2 - (75.3)/(0.400)R_1} \left[\begin{array}{cc|c} 0.400 & 99.6 & 100 \\ 0 & -18700 & -18800 \end{array} \right].$$

Observe that, rounding to three significant digits,

$$\frac{75.3}{0.400}(99.6) = 18749.7 \approx 18700$$

and

$$\frac{75.3}{0.400}(100) = 18825 \approx 18800.$$

So, the numbers -18700 and -18800 in the second row of the reduced matrix are a result of the following rounding (to three significant digits):

$$-45.3 - \frac{75.3}{0.400}(99.6) \approx -45.3 - 18700 = -18745.3 \approx -18700$$

and

$$30.0 - \frac{75.3}{0.400}(100) \approx 30.0 - 18800 = -18770 \approx -18800.$$

Therefore,

$$-18700y = -18800 \implies y = 1.00534759358 \approx 1.01.$$

We now use $y = 1.01$ to find x . We have

$$0.400x + (99.6)(1.01) = 100.$$

So,

$$x = \frac{100}{0.400} - \frac{(99.6)(1.01)}{0.400} = 250 - 251.49 \approx 250 - 251 = -1.00$$

(here we rounded 251.49 to three significant digits).