**What does it mean to have the "habits of mind of a mathematical thinker?"**

*Adding it Up* argues that **mathematical proficiency** has five strands:

- **Conceptual understanding**
  - Comprehension of mathematical concepts, operations, and relations
- **Procedural fluency**
  - Skill in carrying out procedures flexibly, accurately, efficiently, and appropriately
- **Strategic competence**
  - Ability to formulate, represent, and solve mathematical problems
- **Adaptive reasoning**
  - Capacity for logical thought, reflection, explanation, and justification
- **Productive disposition**
  - Habitual inclination to see mathematics as sensible, useful, worthwhile, coupled with a belief in diligence and one's on efficacy.

Hung-Hsi Wu, a Professor at Berkeley proposes five basic characteristics of good mathematics:

- **Precision**: Mathematical statements are clear and unambiguous.
- **Definitions**: They are the bedrock of mathematics. No definitions, no mathematics.
- **Reasoning**: The lifeblood of mathematics. The engine that drives problem solving.
- **Coherence**: Mathematics is a tapestry in which all the concepts and skills are interwoven.
- **Purposefulness**: Mathematics is goal-oriented.

**How do we develop teachers with the habits of mind of a mathematical thinker?**

Two general themes guide *The Mathematical Education of Teachers*:

- the intellectual substance in school mathematics; and
- the special nature of the mathematical knowledge needed for teaching.

Recommendations:

1) Prospective teachers need mathematics courses that develop a deep understanding of the mathematics they will teach.

2) Although the quality of mathematical preparation is more important than the quantity, the following amount of mathematics coursework for prospective teachers is recommended.

   i) Prospective elementary grade teachers (K-4) should be required to take at least 9 semester-hours on fundamental ideas of elementary school mathematics.
   ii) Prospective middle grade teachers of mathematics (5-8) should take at least 21 semester-hours of mathematics, that include at least 12 semester-hours on fundamental ideas of school mathematics appropriate for middle grades teachers.