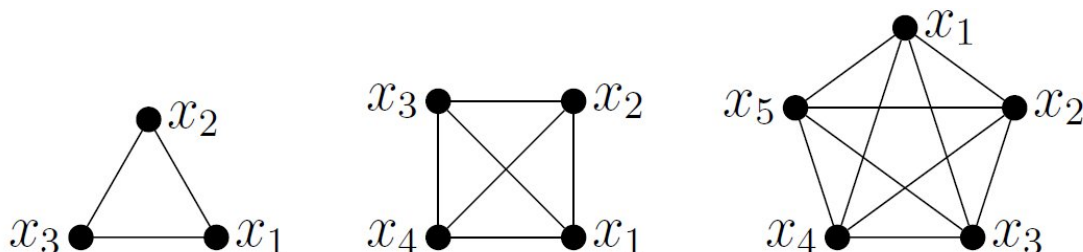


# MATH 4920/8510: Combinatorial Commutative Algebra

## Course Information Sheet and Syllabus

### Winter 2019




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#### BASIC COURSE DETAILS

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**Instructor:** Dr. Susan Cooper

**Class Times and Location:** Tuesdays & Thursdays 11:30 – 12:45 p.m., 376 University College

**Credit Hours:** 3

**Pre-Requisites:** Consent of department.

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#### INSTRUCTOR CONTACT INFORMATION

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**Instructor:** Dr. Susan Cooper

**Email:** susan.cooper@umanitoba.ca

**Office:** Machray Hall, Room 464

**Office Phone:** (204) 474-9701

**Office Hours:** Tuesdays 1:00 – 1:50 p.m. & Thursdays 3:00 – 3:50 p.m.; or by appointment

**Correspondences and Appointments:** The most reliable way to contact me is via email. I will reply to an email within 24 hours of receiving it Mondays – Thursdays; an email received on a Friday will receive a reply the following Monday. All appointments are to be made via email.

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#### COURSE DESCRIPTION AND GOALS

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*“Mathematical science is in my opinion an indivisible whole, an organism whose vitality is conditioned upon the connection of its parts.” — David Hilbert*

**University of Manitoba Course Calendar Description for MATH 4290:** Topics of current interest in Mathematics or Applied Mathematics upon the interests and requirements of students and faculty, and will include specialized topics not available in regular course offerings.

**University of Manitoba Course Calendar Description for MATH 8510:** Designed to accommodate special topics not included in topics courses.

**General Course Description and Goals:** Combinatorial commutative algebra is an area of mathematics which lies at the intersection of commutative algebra and combinatorics. Despite being a young branch of mathematics, it has been growing in strength and popularity quite quickly. One of the main goals of combinatorial commutative algebra is to add flexibility to both commutative algebra

and combinatorics by using and developing techniques within each individual area to solve problems in the other. Developing such a dictionary between the two areas has proven to be quite fruitful and has been influenced by additional fields such as statistics, physics, and polyhedral geometry. Not surprisingly, combinatorial commutative algebra is a thriving and intriguing area of mathematics.

MATH 4920/8510: Combinatorial Commutative Algebra is a one-term course that provides an introduction to combinatorial commutative algebra. We will focus on square-free monomial ideals and finite, simple graphs. Topics that will be considered include: edge and cover ideals, colouring graphs, simplicial complexes, splitting monomial ideals, Föberg's Theorem, associated primes and powers of ideals. Along the way, commutative algebra techniques will be introduced and explored.

By considering concrete examples, you will make conjectures and then try to verify or disprove them. You will gain facility and become confident that you can *do* mathematics and you will experience the joy of discovering hidden patterns and mathematical truths. After successful completion of the course, students will be able to state, prove, apply fundamental theorems, and construct and work with a variety of concrete examples. In addition, students will be well-prepared for follow-up courses in commutative algebra.

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## COURSE MATERIALS

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**Textbook:** *A Beginner's Guide to Edge and Cover Ideals*, by Adam Van Tuyl, in *Monomial Ideals, Computations and Applications*, Lecture Notes in Mathematics Volume 2083, 2013, pp. 63–94 and freely available at

[https://ms.mcmaster.ca/~vantuyl/research/monica\\_school.html](https://ms.mcmaster.ca/~vantuyl/research/monica_school.html)

**Note:** We will be using the textbook as a guide to learning the course material. However, the textbook assumes some background in commutative algebra. As such, we will supplement the course with material not presented in the textbook. Thus the lecture notes are the main resource for this course. Some popular resources that you might find helpful at times for the background commutative algebra include:

- *Cohen-Macaulay Rings* by W. Bruns and J. Herzog (Cambridge Studies in Advanced Mathematics, 39. Cambridge University Press, Cambridge, 1993).
- *Monomial Ideals* by J. Herzog and T. Hibi (Graduate Texts in Mathematics 260, Springer, 2011).
- *Combinatorial Commutative Algebra* by E. Miller and B. Sturmfels (Graduate Texts in Mathematics 227, Springer-Verlag, New York, 2004).
- *Graded Syzygies* by I. Peeva (Algebra and Applications 114, Springer, 2010).
- *Steps in Commutative Algebra* by R. Y. Sharp (2nd edition Cambridge University Press, 2000).
- *Monomial Algebras* by R. H. Villarreal (Monographs and Textbooks in Pure and Applied Mathematics, 238. Marcel Dekker, Inc., New York, 2001.).
- *Introduction to Commutative Algebra* by M. F. Atiyah and I. G. Macdonald (Addison-Wesley Publishing Company Inc., 1969).

**Course Web-Page:** We will use UM Learn and the instructor's web-page which can be found at [http://server.math.umanitoba.ca/~coopers5/courses\\_umanitoba/math4920\\_8510\\_w19.html](http://server.math.umanitoba.ca/~coopers5/courses_umanitoba/math4920_8510_w19.html)

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## ASSESSMENTS AND COURSE GRADES

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**Participation Presentations:** Mathematics is not a spectator sport. The best way to learn mathematics is by doing mathematics! Moreover, the depth to which one truly understands a topic is reflected in how well one can teach the topic to another. Students and I will take turns presenting material. We will follow the notes in the text, but often students will have to go beyond the scope of the text to fill in details for a complete description.

Each student will give a minimum of three consecutive 75-minute lectures. For each lecture, students are to submit lecture notes to be shared with classmates on UM Learn. The lecture notes must provide a list of all resources that were used in preparation of the lectures and must be submitted by 4 p.m. on the day the lecture was presented. A detailed rubric for the presentations and lecture notes will be distributed before the first student presentation. At the very least, presentations and notes will be judged for organization, mathematical content, and delivery. Written feedback will be provided and is meant to *improve* presentation skills along with the correctness of the mathematics presented.

**Problem Sets:** To encourage constant engagement with the course material, each student will submit 1–3 possible homework exercises with solutions based on their lectures for fellow classmates to complete. Presenters are to submit their suggested exercises and solutions by the last day of their series of lectures. Periodically throughout the term, Problem Sets consisting of these exercises will be assigned and collected. It is highly recommended that you carefully read the material and complete all the exercises in a Problem Set well before the deadline in order to keep on track and make sure that you have solid foundations before moving on to the next topic. The Problem Sets are intended to gauge your understanding of the material while presenting opportunities for you to practice the fine art of communicating mathematics. A subset of the solutions will be graded based on correctness, completeness, and quality of exposition (clarity, style/creativity, conciseness, etc.). Partial credit will be awarded whenever possible. Any necessary special grading rules will be provided beforehand. Please note that all feedback is meant to *improve* your mathematical abilities and communication. Moreover, students will receive feedback within two weeks of a Problem Set being assigned.

In many areas of life, we deepen our understanding via discussions with others and a variety of resources. Although you are encouraged to work together on Problem Sets and to discuss ideas with myself, you are expected to submit solutions that are written individually and in your own words: see the section entitled “Academic Integrity” and the handout entitled “Guidelines for Problem Sets” for further expectations and more information.

**MATH 8510 Project:** Students registered in MATH 8510 are required to submit a short paper (5–10 pages, typeset using  $\text{\LaTeX}$ ) on an approved topic. Topics are to be approved no later than Thursday, February 28. Papers are to include an introduction, collection of results from the literature and a list of references used. The papers are due by the beginning of class on Tuesday, April 9.

**Deferred Work:** In general, late work will not be accepted. Problem Sets are to be submitted by the beginning of class on the date indicated on the assignment. Lecture Notes are due by 4 p.m. on days that the lectures are presented. Suggested Problem Set exercises and solutions are to be submitted by the last day of a student’s series of lectures. MATH 8510 projects are due by the beginning of class on April 9. Late acceptances will only be granted for unavoidable, documented circumstances as described below:

Circumstance	Required documentation
Illness or other medical situation	Official note from clinic, hospital, doctor, nurse, or other health care provider
Military service	Official military activation orders
Funeral or other family emergency	Official documentation from newspaper, funeral, or medical official
Sports or other official U of M activity	Official documentation from U of M athletics or activity's faculty adviser

Students who wish to request a late acceptance must contact the instructor within 24 hours of the due date/time (initial email contact is sufficient). Please note that recreational activities do not qualify for deferred work. If you have a pre-existing conflict with an assessment, you are expected to make alternative arrangements *beforehand*.

**Class Attendance and Participation:** Your understanding of the course material will be greatly supported by regular attendance and engagement in class meetings. Although you are expected to attend every class and to fully participate in class discussions, attendance will not be taken or be used in the calculation of course grades. However, you are responsible for any missed material when absent. It is often not easy for students to give presentations and so your support and encouragement via attendance is appreciated. Indeed, you wouldn't want to put a lot of time and energy into preparing for a lecture you are to give and then have no audience – please show respect for your classmates and attend class.

**Evaluation Scheme and Letter Grades:** Final course grades will be determined by the following scheme:

	MATH 4920	MATH 8510
<b>Participation Presentations:</b>	60%	60%
<b>Lecture Notes:</b>	25%	20%
<b>Problem Sets:</b>	15%	10%
<b>Project:</b>	N/A	10%

Below are the minimum cut-off ranges for grades; these may decrease at the instructors' discretion.

Letter Grade	Percentage Out Of 100	Final Grade Point Value
A+	95–100	4.5
A	85–94	4.0
B+	80–84	3.5
B	72–79	3.0
C+	65–71	2.5
C	60–64	2.0
D	50–59	1.0
F	Less than 50	0

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## COURSE SCHEDULE AND IMPORTANT DATES

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Since this is a topics course which is driven by student presentations and interests, a schedule of lectures will be agreed to as a class in the first two weeks of class meetings. Note, however, that the topics schedule determined will be subject to change at the discretion of the instructor and/or based on learning needs of the students (subject to Section 2.8 of ROASS: Responsibilities Of Academic Staff With Regard To Students). For convenience, some important term dates are listed in the table below.

Dates	Importance
January 7	First Class Meeting
February 18–22	No Classes (Louis Riel Day & Winter Term Break)
February 28	MATH 8510 Project Topics Approval Due
March 20	Voluntary Withdrawal Deadline
April 9	MATH 8510 Projects Due By 11:30 a.m.

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## UNIVERSITY AND DEPARTMENT OF MATHEMATICS SUPPORT OFFICES AND POLICIES

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A list (entitled Schedule “A”) of supports available to students, including mathematical support, can be found on the course web-page(s).

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## EXPECTATIONS

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**Recording Class Lectures:** Susan Cooper and the University of Manitoba hold copyright over the course materials, presentations, and lectures which form part of this course. No audio or video recording of lectures or presentations is allowed in any format (including photographs), openly or surreptitiously, in whole or in part without permission. Course materials (both paper and digital) are for the participant’s private study and research.

**Using Copyrighted Material:** Please respect copyright. We will use copyrighted material in this course. I have ensured that the content I use is appropriately acknowledged and is copied in accordance with copyright laws and University guidelines. Copyrighted works, including those created by me, are made available for private study and research and must not be distributed in any format without permission. Do not upload copyrighted works to a learning management system (such as UM Learn), or any website, unless an exception to the *Copyright Act* applies or written permission has been confirmed. For more information, see the University’s Copyright Office web-site at <http://umanitoba.ca/copyright/> or contact [um\\_copyright@umanitoba.ca](mailto:um_copyright@umanitoba.ca).

**Course Technology:** It is the University of Manitoba policy that all technology resources are to be used in a responsible, efficient, ethical and legal manner. During class meetings and for Problem Sets, it is expected that you only use technology for educational purposes and that the only technology used is approved by myself and/or the University of Manitoba Student Accessibility Services. You should not participate in personal direct electronic message/posting activities (such as e-mail, texting, video, social networking, etc.) during scheduled class time – this is not only in your best interests for understanding the course material but is respectful behaviour for your classmates. If you absolutely need to take an expected call, then please use the vibrate mode on your cell phone and leave the classroom before using the phone.

**Class Communication:** You are required to obtain and use your University of Manitoba email account for all communication between yourself and the university. All communication must comply with the Electronic Communication with Students Policy.

**Student Accessibility Services:** The University of Manitoba is committed to providing an accessible academic community. *Students Accessibility Services (SAS)* offers academic accommodation supports and services such as note-taking, interpreting, assistive technology and exam accommodations. Students who have, or think they may have, a disability (e.g. mental illness, learning, medical, hearing, injury-related, visual) are invited to contact SAS to arrange a confidential consultation. Students are welcome to meet with the instructors to discuss the accommodations recommended by SAS.

Student Accessibility Services

<http://umanitoba.ca/student/saa/accessibility/>

520 University Centre

Phone: (204) 474-7423

Email: [Student\\_accessibility@umantioba.ca](mailto:Student_accessibility@umantioba.ca)

**Academic Integrity:** You are expected to be academically honest. This means, for example, providing a list of the people (if any) with whom you worked and providing a list of sources other than the textbook (if any) that you used to complete an assignment. Although you are encouraged to work together, you should never submit anything that you do not understand or is not written in your own words. The following excerpt about Academic Honesty is taken from the Department of Mathematics web-page:

The Department of Mathematics, the Faculty of Science and the University of Manitoba regard acts of academic dishonesty in quizzes, tests, examinations or assignments as serious offences and may assess a variety of penalties depending on the nature of the offence. Acts of academic dishonesty include bringing unauthorized materials into a test or exam, copying from another student, plagiarism and examination personation. Students are advised to read the sections entitled “Academic Integrity” and “Final Examinations” (in particular, the section “Examination Personations”) in the “General Academic Regulations” section of the current Undergraduate Calendar.

Penalties for violation include being assigned a grade of zero on a test or assignment, being assigned a grade of “F” in a course, compulsory withdrawal from a course or program, suspension from a course/program/faculty or even expulsion from the University. For specific details about the nature of penalties that may be assessed upon conviction of an act of academic dishonesty, students are referred to University Policy 1202 (*Student Discipline Bylaw*) and to the Department of Mathematics policy concerning minimum penalties for acts of academic dishonesty. Information regarding cheating and plagiarism is also available from the Faculty of Science web-page.

Students are encouraged to visit the University of Manitoba Academic Integrity site for further information.

**Classroom Atmosphere:** A part of learning is making mistakes. We want to establish a classroom atmosphere where the inevitable false starts and mistakes become an opportunity to improve – not an opportunity for embarrassment. Please be constructive and polite in questioning your colleagues.

**Other Expectations and Tips for Success:** I ask that you have a well-defined sense of professionalism, that you always put forth your best effort, and that you develop a sense of responsibility to your educational community. I ask that you exhibit a persistent desire to learn. In return I will provide you with significant support. Also:

- Be positive, open, and responsive to feedback.
- Be an active participant - mathematics is learned by doing; this includes participating fully in classroom activities (please, turn your cell phones off during class), completing the Problem Sets, participating via Participation Presentations, critically thinking about the mathematics during and outside of class. *In order for this class to be successful, it is imperative that you*

*commit to attending class/tutorial sessions regularly, that you commit to preparing beforehand for class meetings, and that you commit to participating in class meetings!*

- Be/become a “risk taker”.
- Be committed.
- Be patient with yourself - it takes time to master newly learned things. Ask for assistance when it is needed. Constantly try to improve yourself as a mathematician.
- Starting with the first class, study in-depth and regularly.
- It is tempting to just copy available solutions. However, struggling through the exercises on your own is an important phase of the learning process.
- Get help as soon as you need it: ask questions in class and office hours; form a study group with your classmates; read alternate resources.
- Like in all areas of life, constructive feedback can be difficult to digest and accept. Please know that the feedback provided in this course is meant to *improve* your mathematical solutions and communication. Please take the feedback seriously and apply it to your future work.
- Everyone wants you to succeed. Please speak with me regarding any concerns you may have.
- Relax and have fun with the course!