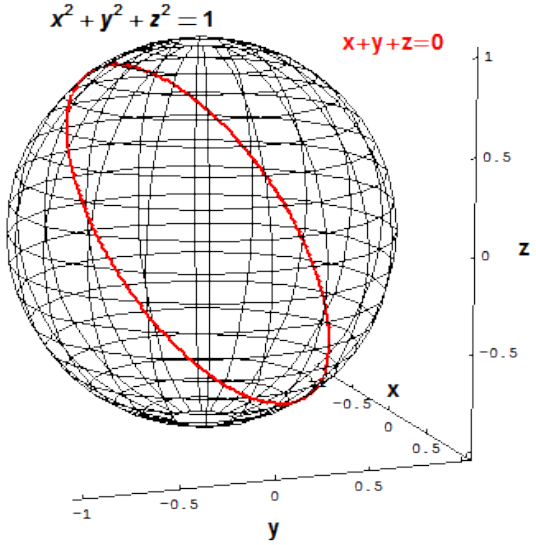
MATH 4340/7340: Introduction To Algebraic Geometry
Course Information Sheet and Syllabus
Winter 2019



BASIC COURSE DETAILS

Instructor: Dr. Susan Cooper

Class Times and Location: Tuesdays & Thursdays 10:00 – 11:15 a.m., 376 University College

Credit Hours: 3

Pre-Requisites:

- MATH 4340: MATH 3322 (C) or the former MATH 3350 (C) or consent of instructor;
- MATH 7340: Permission of department.

Note: MATH 7340 may not be held with MATH 4340.

INSTRUCTOR CONTACT INFORMATION

Instructor: Dr. Susan Cooper **Email:** susan.cooper@umanitoba.ca

Office: Machray Hall, Room 464 **Office Phone:** (204) 474–9701

Office Hours: Tuesdays 1:00 – 1:50 p.m. & Thursdays 3:00 – 3:50 p.m.; or by appointment

Correspondences and Appointments: The most reliable way to contact me is via email. I will reply to an email within 24 hours of receiving it Mondays – Thursdays; an email received on a Friday will receive a reply the following Monday. All appointments are to be made via email.

COURSE DESCRIPTION AND GOALS

"Algebra is but written geometry; geometry is but drawn algebra." — Sophie Germain

University of Manitoba Course Calendar Description: This course will introduce students to the basics of affine and projective varieties through a combination of basic theoretical tools and elementary examples.

General Course Description and Goals: Algebraic geometry is an area of mathematics which focuses on the study of zero point sets of systems of polynomial equations. Techniques used to solve problems from this area often come from commutative algebra. The study of algebraic geometry involves topics that are important branches of mathematics which are thriving and intriguing in their own right. As such, it is not surprising that algebraic geometry hosts many applications in areas such as number theory, complex analysis, and topology (just to name a few!). Moreover, computational algebraic geometry helps us to develop software and algorithms for studying geometric objects.

MATH 4340/7340 is a one-term course that provides an introduction to algebraic geometry, including computational algebraic geometry. We will consider numerous topics such as affine space and varieties, Gröbner bases, elimination theory, Hilbert's Nullstellensatz, Zariski closures, irreducible varieties, polynomial and rational functions on varieties, projective space and varieties, Bezout's Theorem, and the dimension of a variety. At every step of the course, connections to commutative algebra will be highlighted.

By considering concrete examples, you will make conjectures and then try to verify or disprove them. You will gain facility and become confident that you can *do* mathematics and you will experience the joy of discovering hidden patterns and mathematical truths. After successful completion of the course, students will be able to state, prove, apply fundamental theorems, and construct and work with a variety of concrete examples. In addition, students will be well-prepared for follow-up courses in algebraic geometry and commutative algebra.

COURSE MATERIALS

Textbook: *Ideals, Varieties, and Algorithms: An Introduction to Computational Algebraic Geometry and Commutative Algebra*, fourth edition, by David A. Cox, John Little, and Donal O'Shea

Note: At times, we may supplement the course with material not presented in the textbook. Thus the lecture notes are the main resource for this course. One open source resource that you might find helpful at times is:

- *Algebraic Curves: An Introduction to Algebraic Geometry* by William Fulton, available at
  `http://www.math.lsa.umich.edu/~wfulton/CurveBook.pdf`

Course Web-Page: We will use UM Learn and the instructor's web-page which can be found at
`http://server.math.umanitoba.ca/~coopers5/courses_umanitoba/math4340_7340_w19.html`

ASSESSMENTS AND COURSE GRADES

Problem Sets and Readings: Mathematics is not a spectator sport. The best way to learn mathematics is by doing mathematics! To encourage constant engagement with the course material, a collection of exercises and readings will be assigned and collected on a weekly basis. For each Problem Set, students registered in MATH 7340 will be asked to complete more problems than those registered in MATH 4340. It is highly recommended that you carefully read the material and complete all the exercises in a Problem Set well before the deadline in order to keep on track and make sure that you have solid foundations before moving on to the next topic. The Problem Sets are intended to gauge your understanding of the material while presenting opportunities for you to practice the fine art of communicating mathematics. A subset of the solutions will be graded based on correctness, completeness, and quality of exposition (clarity, style/creativity, conciseness, etc.). Partial credit will be awarded whenever possible. Any necessary special grading rules will be provided beforehand. Please note that all feedback is meant to *improve* your mathematical abilities and communication. Moreover, students will receive ample feedback (at least 3 graded Problem Sets) before the Voluntary Withdrawal Deadline.

In many areas of life, we deepen our understanding via discussions with others and a variety of resources. Although you are encouraged to work together on Problem Sets and to discuss ideas with myself, you are expected to submit solutions that are written individually and in your own words: see the section entitled "Academic Integrity" and the handout entitled "Guidelines for Problem Sets" for further expectations and more information.

Participation Presentations: It is important to be able to explain your mathematical ideas to others not just in written form but also via dialogue. Moreover, being able to explain your ideas is an excellent opportunity to really make sure you understand all the intricate details. Students will be required to present 1–3 (depending on level of exercises chosen and if registered in MATH 4340 or MATH 7340) solutions from a Problem Set. The chosen problems and presentation dates must be approved before-hand by me. Students will be given ample time to prepare for these presentations and are most welcome to discuss these with me prior. A detailed rubric will be distributed in the first week of class meetings and will involve three main points: organization, mathematical content, and delivery. Students are highly encouraged to practice their presentation beforehand. Written feedback will be provided and is meant to *improve* presentation skills along with the correctness of the mathematics presented.

Examinations: There will be no mid-term examination administered in this course. However, there will be a take-home final examination. The final examination will be distributed to students on Friday, April 12 and will be due by 4:00 p.m. on Tuesday, April 16. Students registered in MATH 7340 will be required to complete more problems than those registered in MATH 4340. Any necessary special grading rules for the final examination will be announced and discussed prior. As with Problem Sets, solutions to the exercises will be assessed based on correctness, completeness, and quality of exposition. No resources outside the course notes and textbook will be permitted (including working with fellow students).

Deferred Work: In general, late work will not be accepted. Problem Sets are to be submitted by the beginning of class on the date indicated on the assignment. The take-home final examination will be due by 4:00 p.m. on Tuesday, April 16. Late acceptances will only be granted for unavoidable, documented circumstances as described below:

Circumstance	Required documentation
Illness or other medical situation	Official note from clinic, hospital, doctor, nurse, or other health care provider
Military service	Official military activation orders
Funeral or other family emergency	Official documentation from newspaper, funeral, or medical official
Sports or other official U of M activity	Official documentation from U of M athletics or activity's faculty adviser

Students who wish to request a late acceptance must contact the instructor within 24 hours of the due date/time (initial email contact is sufficient). Please note that recreational activities do not qualify for deferred work. If you have a pre-existing conflict with an assessment, you are expected to make alternative arrangements *beforehand*.

Class Attendance and Participation: Your understanding of the course material will be greatly supported by regular attendance and engagement in class meetings. Although you are expected to attend every class and to fully participate in class discussions, attendance will not be taken or be used in the calculation of course grades However, you are responsible for any missed material when absent. If time permits, we will discuss some of the exercises from the Problem Sets. In such discussions, students will be asked to share their ideas. Please take your turn in these activities – it will greatly improve your understanding of the material. In particular, if you are absent from class then you will miss the opportunity to learn from your classmates.

Evaluation Scheme and Letter Grades: Final course grades will be determined by the following scheme:

Task	**Percentage of Grade**
Problem Sets	60%
Participation Presentations	10%
Take-Home Final Examination	30%

Below are the minimum cut-off ranges for grades; these may decrease at the instructors' discretion.

Letter Grade	**Percentage Out Of 100**	**Final Grade Point Value**
A+	93–100	4.5
A	86–92	4.0
B+	80–85	3.5
B	73–79	3.0
C+	67–72	2.5
C	60–66	2.0
D	50–59	1.0
F	Less than 50	0

COURSE SCHEDULE AND IMPORTANT DATES

The topics schedule below is subject to change at the discretion of the instructor and/or based on learning needs of the students (subject to Section 2.8 of Responsibilities Of Academic Staff With Regard To Students). Assessment dates will not change.

Dates	Topics	Problem Set Due
Jan. 7–11	affine varieties, parametrizations, polynomials, ideals	
Jan. 14–18	Gröbner bases	# 1 Jan. 17
Jan. 21–25	Gröbner bases	# 2 Jan. 24
Jan. 28–Feb. 1	Elimination Theory	# 3 Jan. 31
Feb. 4–8	Elimination Theory	# 4 Feb. 7
Feb. 11–15	Hilbert's Nullstellensatz, radical ideals, varieties, ideal operations	# 5 Feb. 14
No Classes: February 18–22 (Louis Riel Day & Winter Term Break)		
Feb. 25–March 1	irreducible varieties decompositions of varieties	# 6 Feb. 28
March 4–8	polynomial and rational functions on a variety	# 7 March 7
March 11–15	polynomial and rational functions on a variety	# 8 March 14
March 18–22	projective varieties	# 9 March 21
Voluntary Withdrawal Deadline: Wednesday, March 20		
March 25–29	projective elimination theory, quadric hypersurfaces Bezout's Theorem	# 10 March 28
April 1–5	dimension of a variety	# 11 April 4
April 9	dimension of a variety	
Take-Home Final Examination Assigned: April 12		
Take-Home Final Examination Due: April 16 at 4 p.m.		

UNIVERSITY AND DEPARTMENT OF MATHEMATICS SUPPORT OFFICES AND POLICIES

A list (entitled Schedule "A") of supports available to students, including mathematical support, can be found on the course web-page(s).

EXPECTATIONS

Recording Class Lectures: Susan Cooper and the University of Manitoba hold copyright over the course materials, presentations, and lectures which form part of this course. No audio or video recording of lectures or presentations is allowed in any format (including photographs), openly or surreptitiously, in whole or in part without permission. Course materials (both paper and digital) are for the participant's private study and research.

Using Copyrighted Material: Please respect copyright. We will use copyrighted material in this course. I have ensured that the content I use is appropriately acknowledged and is copied in accordance with copyright laws and University guidelines. Copyrighted works, including those created by me, are made available for private study and research and must not be distributed in any format without permission. Do not upload copyrighted works to a learning management system (such as UM Learn), or any website, unless an exception to the *Copyright Act* applies or written permission has been confirmed. For more information, see the University's Copyright Office web-site at `http://umanitoba.ca/copyright/` or contact `um_copyright@umanitoba.ca`.

Course Technology: It is the University of Manitoba policy that all technology resources are to be used in a responsible, efficient, ethical and legal manner. During class meetings and for Problem Sets and the take-home final examination, it is expected that you only use technology for educational purposes and that the only technology used is approved by myself and/or the University of Manitoba Student Accessibility Services. You should not participate in personal direct electronic message/posting activities (such as e-mail, texting, video, social networking, etc.) during scheduled class time – this is not only in your best interests for understanding the course material but is respectful behaviour for your classmates. If you absolutely need to take an expected call, then please use the vibrate mode on your cell phone and leave the classroom before using the phone.

Class Communication: You are required to obtain and use your University of Manitoba email account for all communication between yourself and the university. All communication must comply with the Electronic Communication with Students Policy.

Student Accessibility Services: The University of Manitoba is committed to providing an accessible academic community. *Students Accessibility Services (SAS)* offers academic accommodation supports and services such as note-taking, interpreting, assistive technology and exam accommodations. Students who have, or think they may have, a disability (e.g. mental illness, learning, medical, hearing, injury-related, visual) are invited to contact SAS to arrange a confidential consultation. Students are welcome to meet with the instructors to discuss the accommodations recommended by SAS.

Student Accessibility Services
`http://umanitoba.ca/student/saa/accessibility/`
520 University Centre
Phone: (204) 474–7423
Email: Student_accessibility@umantioba.ca

Academic Integrity: You are expected to be academically honest. This means, for example, providing a list of the people (if any) with whom you worked and providing a list of sources other than the textbook (if any) that you used to complete an assignment. Although you are encouraged to work together, you should never submit anything that you do not understand or is not written in your own words. The following excerpt about Academic Honesty is taken from the Department of Mathematics web-page:

The Department of Mathematics, the Faculty of Science and the University of Manitoba regard acts of academic dishonesty in quizzes, tests, examinations or assignments as serious offences and may assess a variety of penalties depending on the nature of the offence. Acts of academic dishonesty include bringing unauthorized materials into a test or exam, copying from another student, plagiarism and

examination personation. Students are advised to read the sections entitled "Academic Integrity" and "Final Examinations" (in particular, the section "Examination Personations") in the "General Academic Regulations" section of the current Undergraduate Calendar.

Penalties for violation include being assigned a grade of zero on a test or assignment, being assigned a grade of "F" in a course, compulsory withdrawal from a course or program, suspension from a course/program/faculty or even expulsion from the University. For specific details about the nature of penalties that may be assessed upon conviction of an act of academic dishonesty, students are referred to University Policy 1202 (*Student Discipline Bylaw*) and to the Department of Mathematics policy concerning minimum penalties for acts of academic dishonesty. Information regarding cheating and plagiarism is also available from the Faculty of Science web-page.

Students are encouraged to visit the University of Manitoba Academic Integrity site for further information.

Classroom Atmosphere: A part of learning is making mistakes. We want to establish a classroom atmosphere where the inevitable false starts and mistakes become an opportunity to improve – not an opportunity for embarrassment. Please be constructive and polite in questioning your colleagues.

Other Expectations and Tips for Success: I ask that you have a well-defined sense of professionalism, that you always put forth your best effort, and that you develop a sense of responsibility to your educational community. I ask that you exhibit a persistent desire to learn. In return I will provide you with significant support. Also:

- Be positive, open, and responsive to feedback.
- Be an active participant - mathematics is learned by doing; this includes participating fully in classroom activities (please, turn your cell phones off during class), completing the Problem Sets, participating via Problem Set Presentations, critically thinking about the mathematics during and outside of class. *In order for this class to be successful, it is imperative that you commit to attending class/tutorial sessions regularly, that you commit to preparing beforehand for class meetings, and that you commit to participating in class meetings!*
- Be/become a "risk taker".
- Be committed.
- Be patient with yourself - it takes time to master newly learned things. Ask for assistance when it is needed. Constantly try to improve yourself as a mathematician.
- Starting with the first class, study in-depth and regularly.
- It is tempting to just copy available solutions. However, struggling through the exercises on your own is an important phase of the learning process.
- Get help as soon as you need it: ask questions in class and office hours; form a study group with your classmates; read alternate resources.
- Like in all areas of life, constructive feedback can be difficult to digest and accept. Please know that the feedback provided in this course is meant to *improve* your mathematical solutions and communication. Please take the feedback seriously and apply it to your future work.
- Everyone wants you to succeed. Please speak with me regarding any concerns you may have.
- Relax and have fun with the course!