

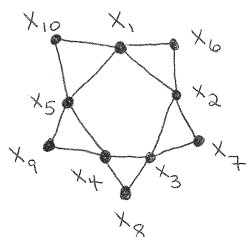
Problem Set 4

Due: 11:30 a.m. on Thursday, March 21

Instructions: All students except for the presenter are to complete all of the exercises below. Be sure to adhere to the expectations outlined on the sheet *Guidelines for Problem Sets*. Submit your solutions in-class or to Dr. Cooper's mailbox in the Department of Mathematics.

Exercises:

1. Let G be the following graph:



- (a) Find $\text{Ass}(J(G^2)) \setminus \text{Ass}(J(G))$.
 - (b) Use part (a) to prove that G is not critically s -chromatic for any $s \in \mathbb{Z}^+$.
 - (c) Let $\mathbf{a} = (400, 400, 400, 400, 400, 200, 200, 200, 200, 200) \in \mathbb{N}^{10}$. Prove that $G^{\mathbf{a}}$ does or does not contain a perfect matching. Do *not* draw $G^{\mathbf{a}}$.
2. Give an example of a non-trivial ideal in a Noetherian ring which has the persistence property. Prove that your example indeed has the persistence property.
 3. A graph G is called *disconnected* if it is possible to partition $V(G)$ into two sets A and B such that there is no edge of G with one vertex in A and the other in B .
Prove that if G is disconnected then G is not vertex critical.