Problem Set 2
Due: 11:30 a.m. on Tuesday, February 12

Instructions: All students except for the presenter are to complete all of the exercises below. Be sure to adhere to the expectations outlined on the sheet *Guidelines for Problem Sets*. Submit your solutions in-class or to Dr. Cooper's mailbox in the Department of Mathematics.

Exercises: All ideals are assumed to be ideals of a commutative ring with identity.

1. Let I, J and K be ideals of a ring R. Prove:
   (a) $(I : J)J \subseteq I$;
   (b) $((I : J) : K) = (I : JK) = ((I : K) : J)$.
2. Let I and J be ideals of a ring R. Prove:
   (a) $r(r(I)) = r(I)$;
   (b) $I \subseteq r(I)$;
   (c) $r(I + J) = r(r(I) + r(J))$.
3. Show that in the polynomial ring $\mathbb{Z}[t]$, the ideal $M = (2, t)$ is maximal, and the ideal $Q = (4, t)$ is M-primary, but is not a power of M.