MATH 2020: Algebra 1 **Dr. S. Cooper, Winter 2018**

Quiz 6
Sample Solutions

Name: _____

Student Number: _____

In the space provided, please write your solutions to the following exercises. *Fully explain your reasoning.* Remember to use good notation and full sentences.

Good Luck!

1. Let H be a subgroup of the group G.

   (a) Complete the following definitions:

   (i) The *left coset of H with representative $g \in G$* is

   Solution: $gH = \{gh \mid h \in H\}$.

   (ii) The *index* of H in G is

   Solution: the number of (distinct) left cosets of H in G, denoted $[G : H]$.

   (b) If $ghg^{-1} \in H$ for all $g \in G, h \in H$, show that right cosets are identical to left cosets. That is, show that $gH = Hg$ for all $g \in G$.

   Solution: Let $a \in gH$. Then $a = gh$ for some $h \in H$. Note that

   $$a = gh = ghg^{-1}g = (ghg^{-1})g \in Hg.$$

   Thus, $a \in Hg$ and so $gH \subset Hg$.

   On the other hand, let $b \in Hg$. Then $b = h_1 g$ for some $h_1 \in H$. Thus,

   $$b = gg^{-1}h_1 g = g(g^{-1}h_1 g).$$

   Let $x = g^{-1} \in G$. Then, by substitution and our assumption,

   $$b = g(xh_1 x^{-1}) \in gH$$

   and so $Hg \subset gH$.

   Therefore, $gH = Hg$ as desired.

2. Suppose that G is a finite group with 60 elements. What are the orders of possible subgroups of G?

   Solution: By Lagrange's Theorem, the order of any subgroup of G must divide 60. Thus, the set of orders of possible subgroups of G is $\{1, 2, 3, 4, 5, 6, 10, 12, 15, 20, 30, 60\}$.