MATH 2020: Algebra 1 **Dr. S. Cooper, Winter 2018**

Quiz 1
Sample Solutions

Name: ____________________

Student Number: ____________________

In the space provided, please write your solution to the following exercises. *Fully explain your reasoning.* Remember to use good notation and full sentences.

Good Luck!

1. Let A and B be sets in the universal set $\mathcal{U}$. Complete the following definitions:

   (i) The *complement* of A is

   Solution: $A' = \{x \mid x \in \mathcal{U} \text{ and } x \notin A\}$.

   (ii) The *union* of A and B is

   Solution: $A \cup B = \{x \mid x \in A \text{ or } x \in B\}$.

2. Let A and B be sets. Prove $(A \cap B)' = A' \cup B'$.

   Solution: We want to show that $(A \cap B)' \subset A' \cup B'$ and $A' \cup B' \subset (A \cap B)'$. Towards the first inclusion, let $x \in (A \cap B)'$. Then $x \notin A \cap B$. Thus, either $x \notin A$ or $x \notin B$ (or both). Hence, by the definition of A' and B', we have that $x \in A'$ or $x \in B'$ and so $x \in A' \cup B'$. This shows that $(A \cap B)' \subset A' \cup B'$.

   To show the reverse inclusion, let $x \in A' \cup B'$. Then $x \in A'$ or $x \in B'$, and so $x \notin A$ or $x \notin B$. Therefore, x cannot be an element of $A \cap B$. That is, $x \notin A \cap B$ and so $x \in (A \cap B)'$ (by definition of set complement). This shows that $A' \cup B' \subset (A \cap B)'$.

   Since $(A \cap B)' \subset A' \cup B'$ and $A' \cup B' \subset (A \cap B)'$, we conclude that $(A \cap B)' = A' \cup B'$ as desired.