MATH 2020: Algebra 1
Tutorial 9 Worksheet – March 19, 2018

Question 1. Prove whether or not each of the following maps is a homomorphism? If the map is a homomorphism, what is its kernel?

(a) $\phi : \mathbb{R}^* \to GL_2(\mathbb{R})$ defined by

$$\phi(a) = \begin{pmatrix} 1 & 0 \\ 0 & a \end{pmatrix}$$

(b) $\phi : \mathbb{R} \to GL_2(\mathbb{R})$ defined by

$$\phi(a) = \begin{pmatrix} 1 & a \\ 1 & 0 \end{pmatrix}$$

(c): $\phi : GL_2(\mathbb{R}) \to \mathbb{R}$ defined by

$$\phi\left(\begin{pmatrix} a & b \\ c & d \end{pmatrix}\right) = a + d$$

(d): $\phi : GL_2(\mathbb{R}) \to \mathbb{R}^*$ defined by

$$\phi\left(\begin{pmatrix} a & b \\ c & d \end{pmatrix}\right) = ad - bc$$

(e) $\phi : \mathbb{M}_2(\mathbb{R}) \to \mathbb{R}$ defined by

$$\phi\left(\begin{pmatrix} a & b \\ c & d \end{pmatrix}\right) = b$$

Note: Take care in recognizing the appropriate group operations.

Question 2. Let A be an $m \times n$ matrix. Show that matrix multiplication, $x \mapsto Ax$, defines a homomorphism $\phi : \mathbb{R}^n \to \mathbb{R}^m$.

Question 3. If G is an abelian group and $n \in \mathbb{N}$, show that $\phi : G \to G$ defined by $g \mapsto g^n$ is a group homomorphism.

Question 4. If $\phi : G \to H$ is a group homomorphism and G is abelian, prove that $\phi(G)$ is also abelian.

Question 5. If $\phi : G \to H$ is a group homomorphism and G is cyclic, prove that $\phi(G)$ is also cyclic.

Question 6. Show that a homomorphism defined on a cyclic group is completely determined by its action on the generator of the group.