MATH 2020: Algebra 1
Tutorial 8 Worksheet – March 12, 2018

Question 1. For each of the following groups G, determine whether H is a normal subgroup of G. If H is a normal subgroup, write out a Cayley table for the factor group G/H.

(a): $G = A_5$ and $H = \{(1), (123), (132)\}$.

(b): $G = S_4$ and $H = D_4$.

(c): $G = Q_8$ and $H = \{1, -1, I, -I\}$.

Question 2. Let H and K be normal subgroups of H. Show that the intersection $N = H \cap K$ is a normal subgroup of G.

Question 3. If G is an abelian group with normal subgroup H, show that G/H must also be abelian.

Question 4. Prove or disprove: If H is a normal subgroup of G such that H and G/H are abelian, then G is abelian.

Question 5. Prove or disprove: If H and G/H are cyclic, then G is cyclic.

Question 6. Let H be a subgroup of index 2 in a group G. Prove that H must be a normal subgroup of G. Conclude that S_n is not simple for $n \geq 3$.

Question 7. If a group G has exactly one subgroup H of order k, prove that H is normal in G.

Question 8. Define the *centralizer* of an element g in a group G to be the set

$$C(g) = \{x \in G \mid xg = gx\}.$$

Show that $C(g)$ is a subgroup of G. If g is a generator of a normal subgroup of G, prove that $C(g)$ is normal in G.