MATH 2020: Algebra 1
Tutorial 6 Worksheet − February 26, 2018

Question 1. Use Fermat's Little Theorem to show that if $p = 4n + 3$ is prime, there is no solution to the equation $x^2 \equiv -1 \pmod{p}$.

Question 2. If $ghg^{-1} \in H$ for all $g \in G$ and $h \in H$, show that right cosets are identical to left cosets. That is, show that $gH = Hg$ for all $g \in G$.

Question 3. Let G be a group and H a subgroup of G. Prove whether relation $\phi : L_H \to R_H$ from the right cosets to the left cosets given by $\phi(gH) = Hg$ for $g \in G$ is a mapping or not.

Note: This is problem 13 in section 6.4 of the textbook rephrased.

Question 4. Suppose that $[G : H] = 2$. If a and b are not in H, show that $ab \in H$.

Question 5. If $[G : H] = 2$, prove that $gH = Hg$ for all $g \in G$.

Question 6. Suppose that $g^n = e$. Prove that the order of g divides n.

Question 7. Suppose that G is a finite group with 60 elements. What are the orders of possible subgroups of G?

Question 8. Let H be a subgroup of a group G and suppose that $g_1, g_2 \in G$. Prove that the following conditions are equivalent.

(a) $g_1 H = g_2 H$
(b) $Hg_1^{-1} = Hg_2^{-1}$
(c) $g_1 H \subset g_2 H$
(d) $g_2 \in g_1 H$
(e) $g_1^{-1} g_2 \in H$