MATH 2020: Algebra 1
Tutorial 4 Worksheet – January 29, 2018

Question 1. Let a and b be elements of a group G. If $a^4b = ba$ and $a^3 = e$, prove that $ab = ba$.

Question 2. Let G be the group of 2×2 real valued matrices under addition and

$$H = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} : a + d = 0 \right\}.$$

Prove that H is a subgroup of G.

Question 3. Prove or disprove: $SL_2(\mathbb{Z})$, the set of 2×2 matrices with integer entries and determinant one, is a subgroup of $SL_2(\mathbb{R})$.

Question 4. Prove or disprove: If H and K are subgroups of a group G, then $H \cup K$ is a subgroup of G.

Question 5. Let G be a group and $g \in G$. Show that

$$Z(G) = \{x \in G \mid gx = xg \text{ for all } g \in G\}$$

is a subgroup of G. This subgroup is called the *center* of G.

Question 6. Let G be a group and suppose that $(ab)^2 = a^2b^2$ for all $a, b \in G$. Prove that G is an abelian group.

Question 7. Suppose that G is a group and let $a, b \in G$. Prove that if $|a| = m$ and $|b| = n$ with $\gcd(m, n) = 1$, then $\langle a \rangle \cap \langle b \rangle = \{e\}$.

Question 8. Prove that if G has no proper nontrivial subgroups then G is a cyclic group.

Question 9. Let p be prime and r a positive integer. How many generators does $\mathbb{Z}_{p^r}$ have?