MATH 2020: Algebra 1
Tutorial 11 Worksheet – April 2, 2018

Question 1. Find all homomorphisms $\phi : \mathbb{Z}/6\mathbb{Z} \to \mathbb{Z}/15\mathbb{Z}$.

Question 2. Prove that $\mathbb{R}$ is not isomorphic to $\mathbb{C}$.

Question 3. Prove or disprove: The ring

$$\mathbb{Q}(\sqrt{2}) = \{a + b\sqrt{2} \mid a, b \in \mathbb{Q}\}$$

is isomorphic to the ring

$$\mathbb{Q}(\sqrt{3}) = \{a + b\sqrt{3} \mid a, b \in \mathbb{Q}\}.$$

Question 4. Define a map $\phi : \mathbb{C} \to \mathbb{M}_2(\mathbb{R})$ by

$$\phi(a + bi) = \begin{pmatrix} a & b \\ -b & a \end{pmatrix}.$$

Show that ϕ is an isomorphism of $\mathbb{C}$ with $\phi(\mathbb{C})$.

Question 5. If R is a field show that the only two ideals of R are $\{0\}$ and R itself.

Question 6. Let $\phi : R \to S$ be a ring homomorphism. Prove that if R is a commutative ring, then $\phi(R)$ is a commutative ring. (You may assume that $\phi(R)$ is indeed a ring.)

Question 7. Let $\{I_\alpha\}_{\alpha \in A}$ be a collection of ideals in a ring R. Prove that $\cap_{\alpha \in A} I_\alpha$ is also an ideal in R. Give an example to show that if I_1 and I_2 are ideals in R, then $I_1 \cup I_2$ may not be an ideal.

Question 8. Let R be an integral domain. Show that if the only ideals in R are $\{0\}$ and R itself, then R must be a field.