MATH 2020: Algebra 1
Tutorial 1 Worksheet – January 8, 2018

Question 1. Let $f : A \to B$ and $g : B \to C$ be invertible mappings; that is, mappings such that f^{-1} and g^{-1} exist. Show that $(g \circ f)^{-1} = f^{-1} \circ g^{-1}$.

Question 2. Define a relation on $\mathbb{R}^2 \backslash \{(0,0)\}$ by letting $(x_1, y_1) \sim (x_2, y_2)$ if there exists a nonzero real number λ such that $(x_1, y_1) = (\lambda x_2, \lambda y_2)$. Prove that $\sim$ defines an equivalence relation on $\mathbb{R}^2 \backslash \{(0,0)\}$.

Question 3. Which of the following relations $f : \mathbb{Q} \to \mathbb{Q}$ define a mapping? In each case, supply a reason why or why not f is a mapping.

(a) $\quad f(p/q) = \dfrac{p+1}{p-2}$
(b) $\quad f(p/q) = \dfrac{p+q}{q^2}$
(c) $\quad f(p/q) = \dfrac{3p^2}{7q^2} - \dfrac{p}{q}$

Question 4. Prove $(A \cap B)' = A' \cup B'$.

Question 5. Let $f : X \to Y$ be a map with $A_1, A_2 \subset X$ and $B_1, B_2 \subset Y$. Prove that $f(A_1 \cap A_2) \subset f(A_1) \cap f(A_2)$. Give an example in which equality fails.

Question 6. Define a function $f : \mathbb{N} \to \mathbb{N}$ that is

(a) one-to-one but not onto.
(b) onto but not one-to-one.

Question 7. Prove that $A \subset B$ if and only if $A \cap B = A$.

Question 8. Determine whether or not the following relations are equivalence relations on the given set. If the relation is an equivalence relation, then prove this and describe the partitions given by it. If the relation is not an equivalence relation, state why it fails to be one via a specific counter-example.

(a) $m \sim n$ in $\mathbb{Z}$ if $mn > 0$
(b) $m \sim n$ in $\mathbb{Z}$ if $m \equiv n \pmod 6$
(c) $f(x) \sim g(x)$ in the set of all real-valued differentiable functions if $f'(x) = g'(x)$