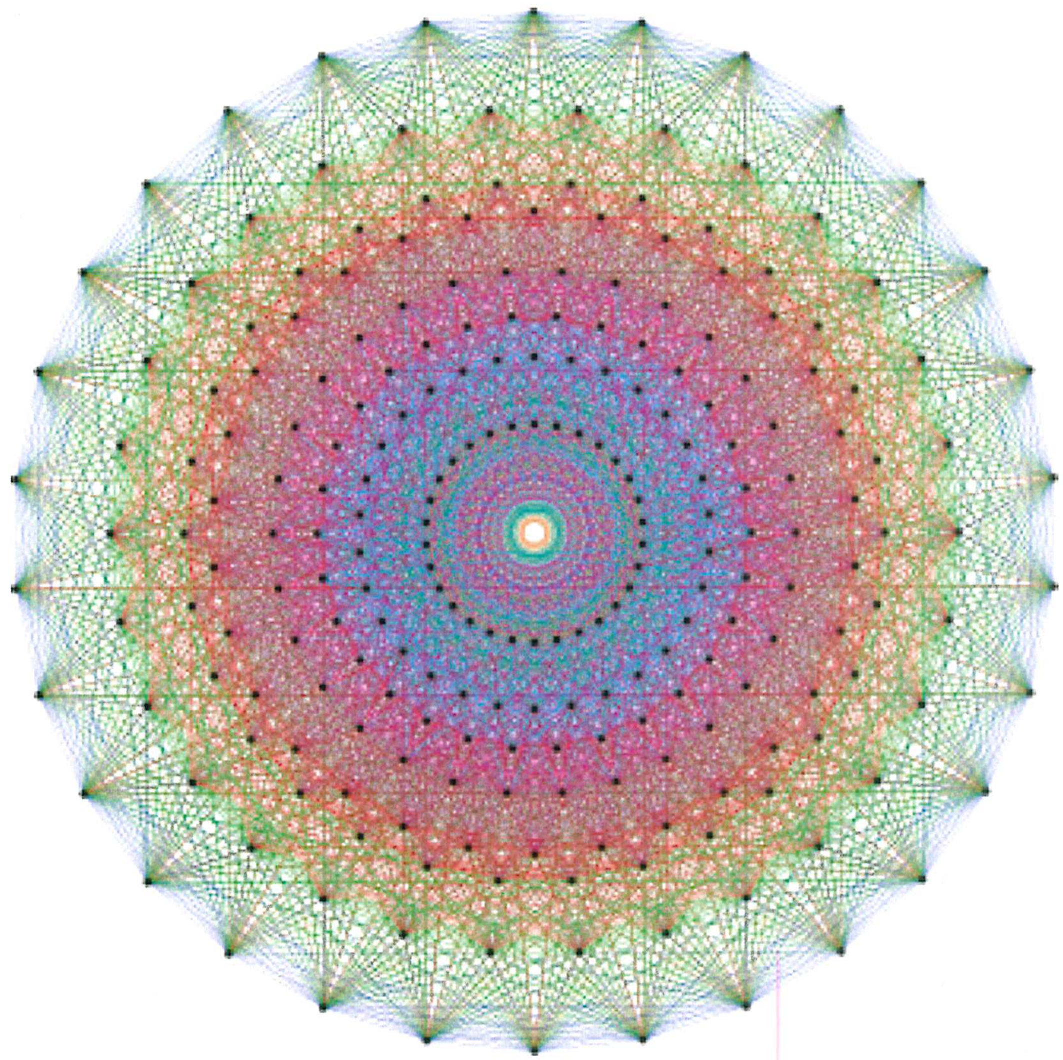
The Beauty In Symmetry



Transformations - Definitions (page 33 of text)

- A **transformation** of the points in the plane is a

- If no two points are moved to a single position, then we say that the transformation is

- A transformation is **onto** if all the positions in the plane are achieved by some points in the rearrangement.

- A **bijection** is a transformation that is both

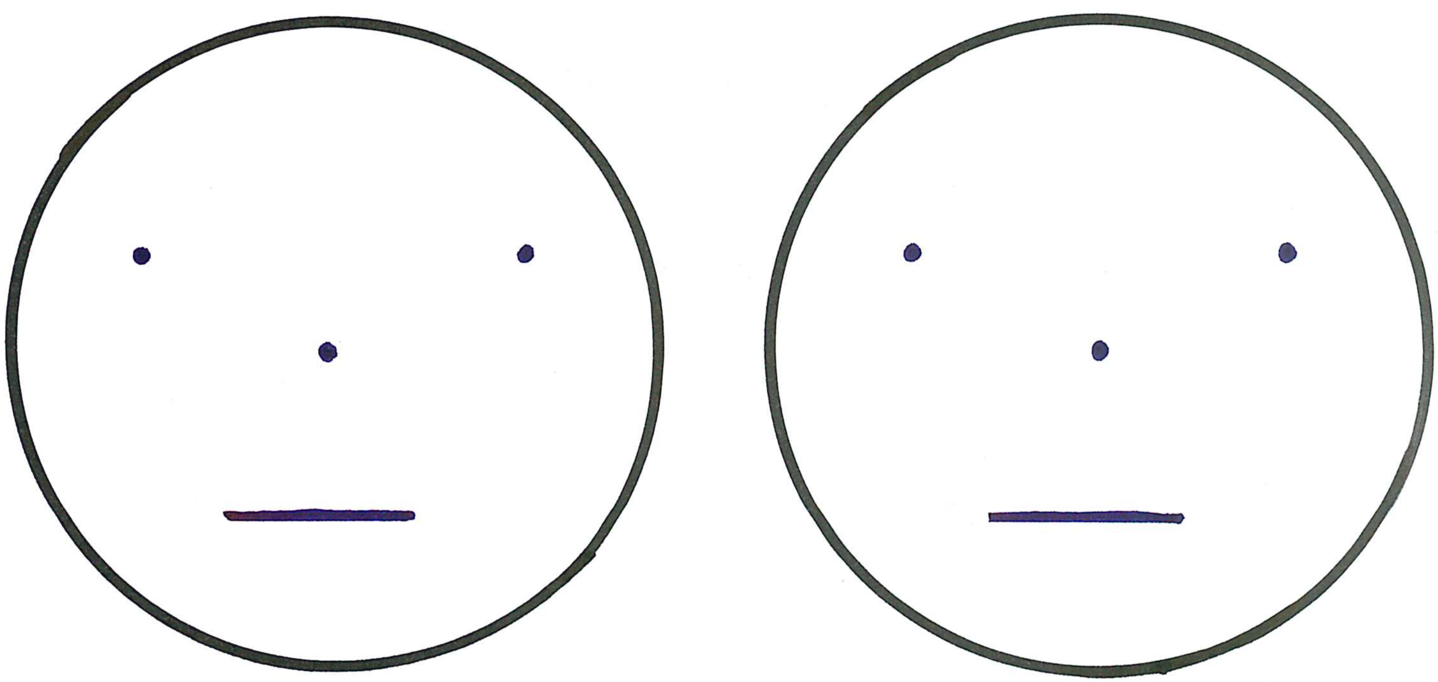
Symmetries Definition (page 33 of text)

- A transformation is **rigid** if it preserves

- Rigid transformations are called

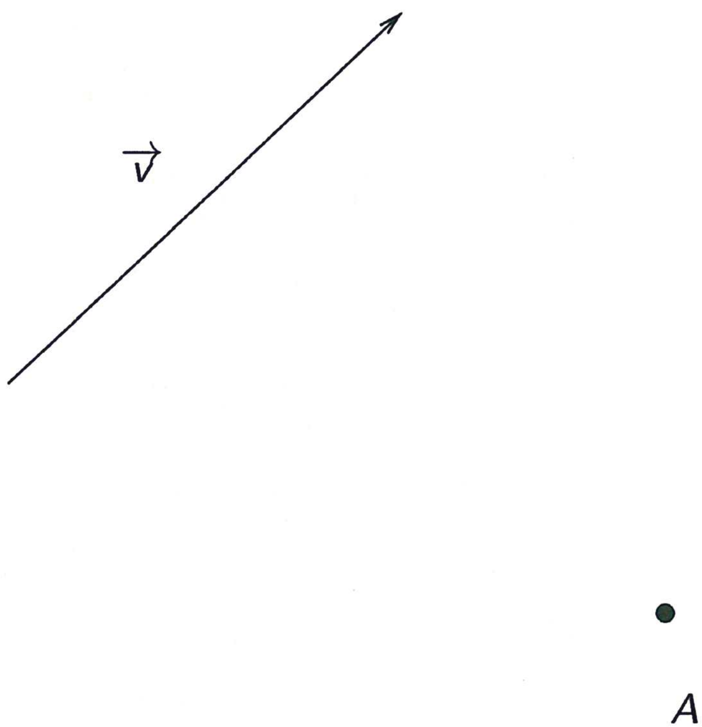
Translations

A **translation** is defined by a vector $\overrightarrow{v}$ and is denoted $f = trans(\overrightarrow{v})$.



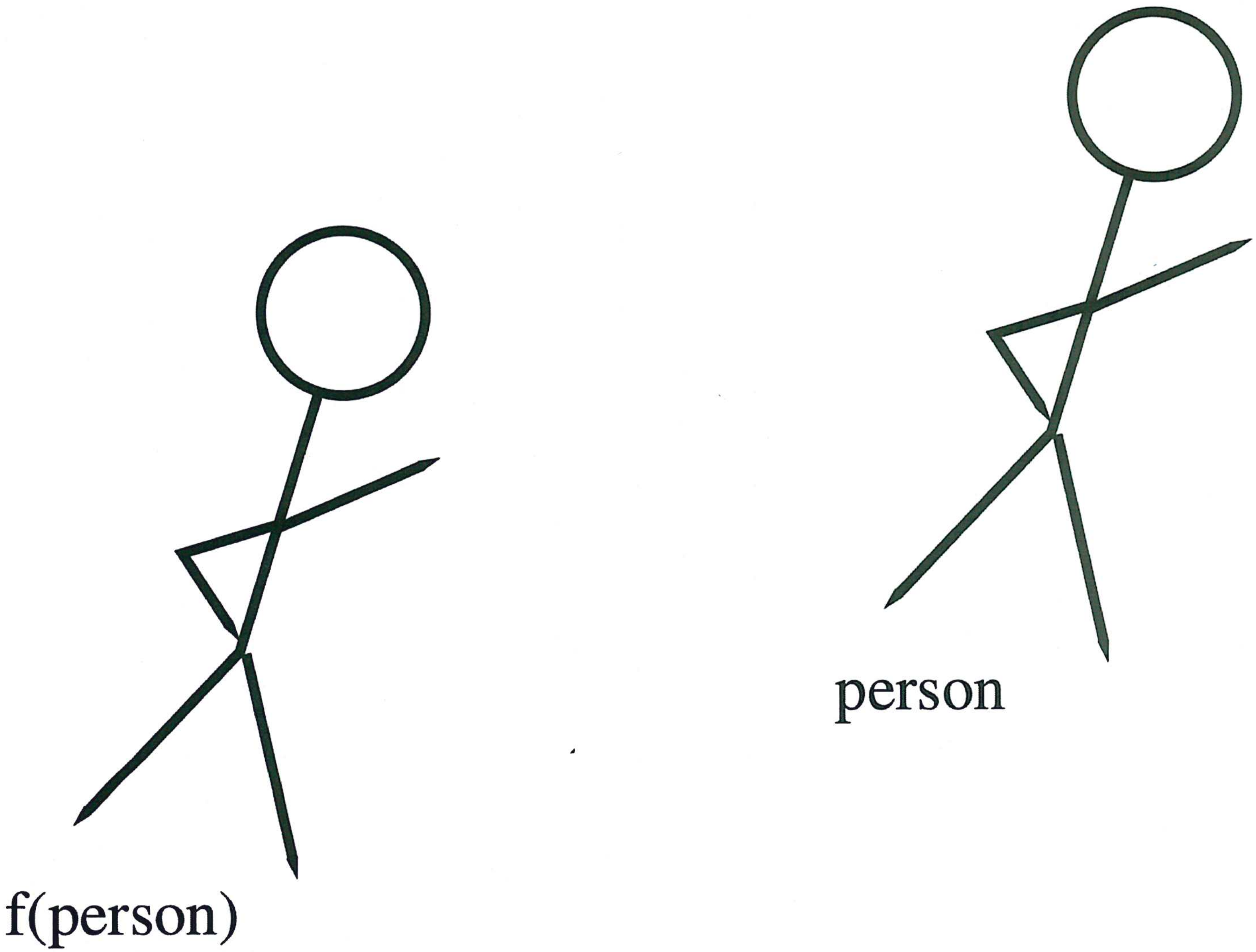
Example

Find the image of A under the symmetry $f = trans(\overrightarrow{v})$.



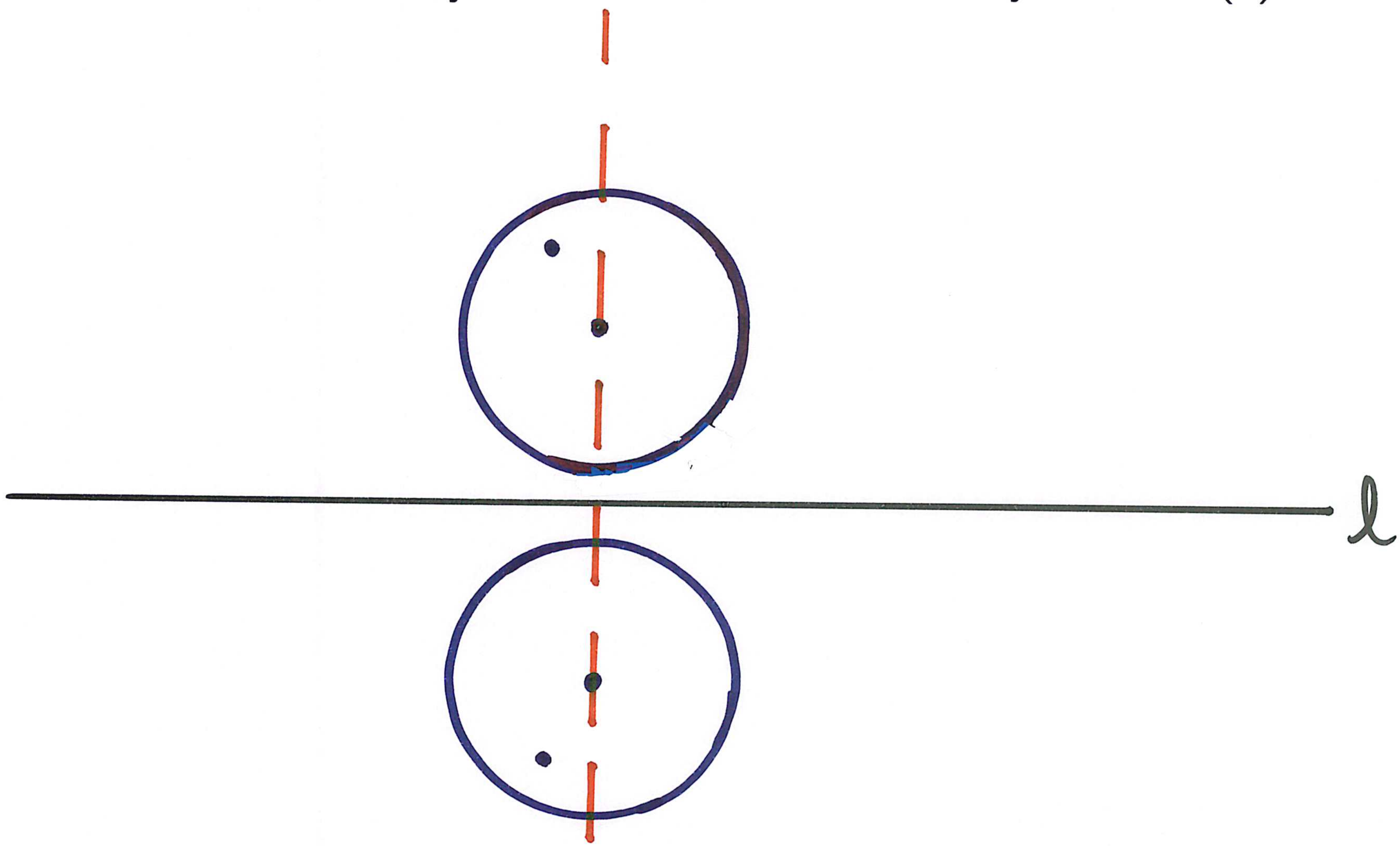
Example

Find the vector of translation $\overrightarrow{v}$ of the symmetry $f = trans(\overrightarrow{v})$.



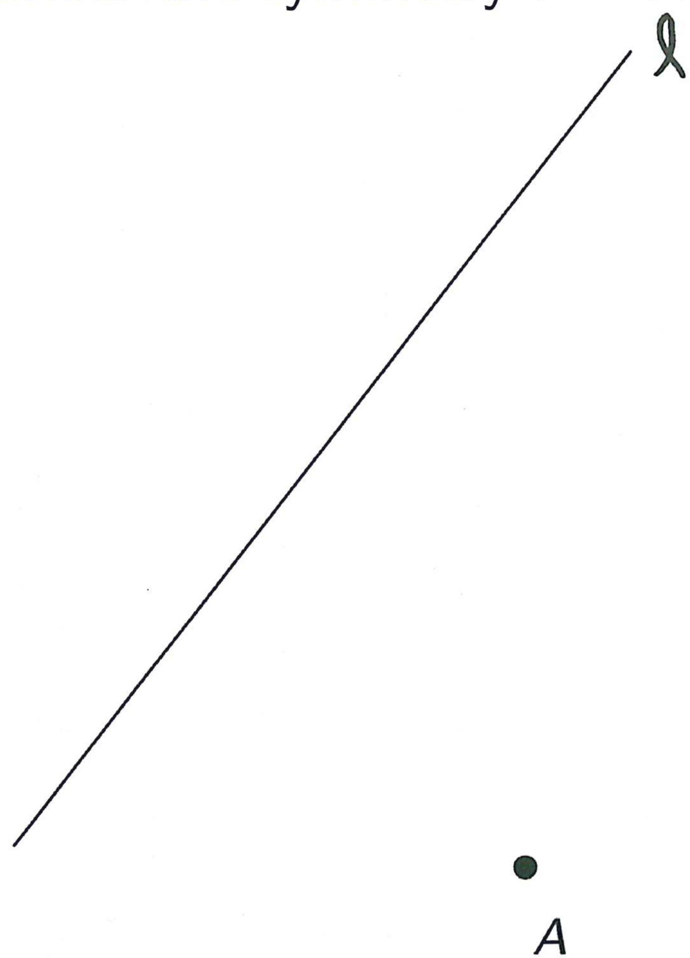
Reflections

A **reflection** is defined by a line ℓ and is denoted by $f = \mathit{refl}(\ell)$.



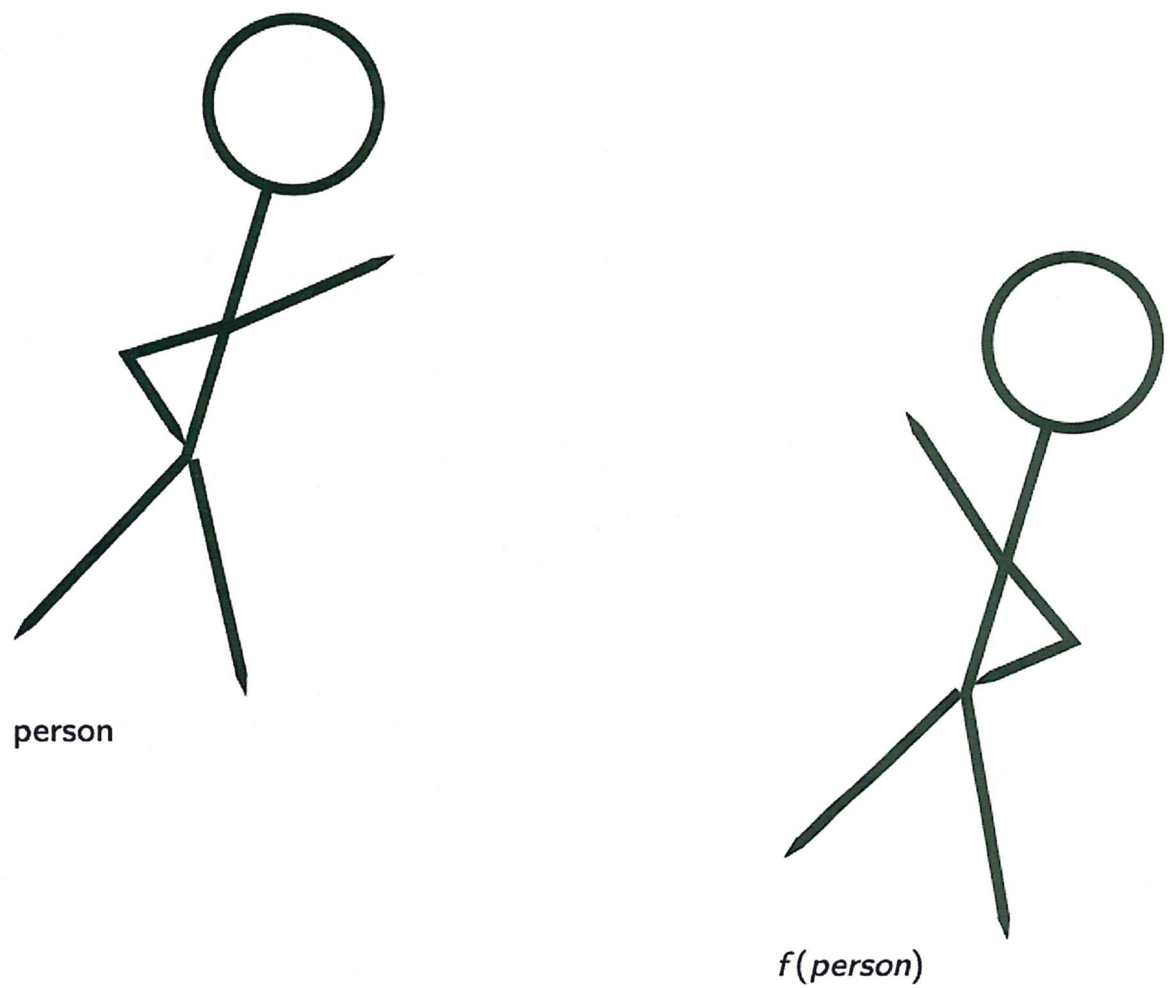
Example

Find the image of A under the symmetry $f = \mathit{refl}(\ell)$.



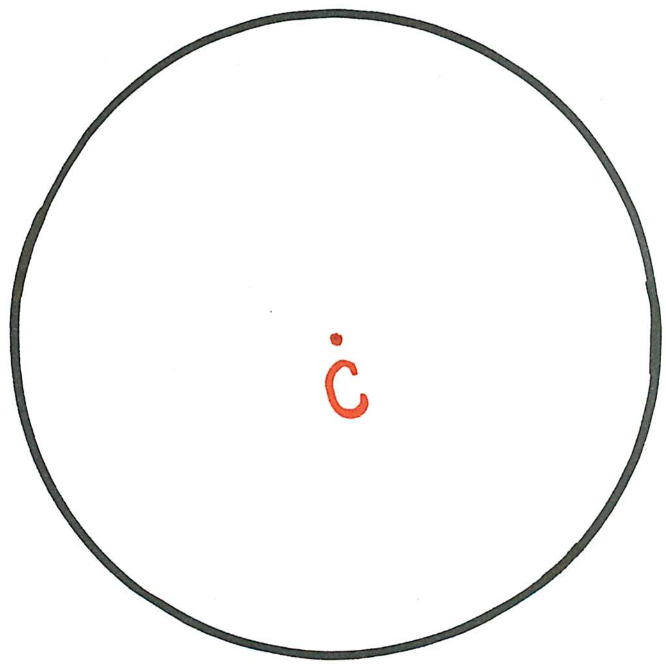
Example

Find the line of reflection ℓ of the symmetry $f = refl(\ell)$.



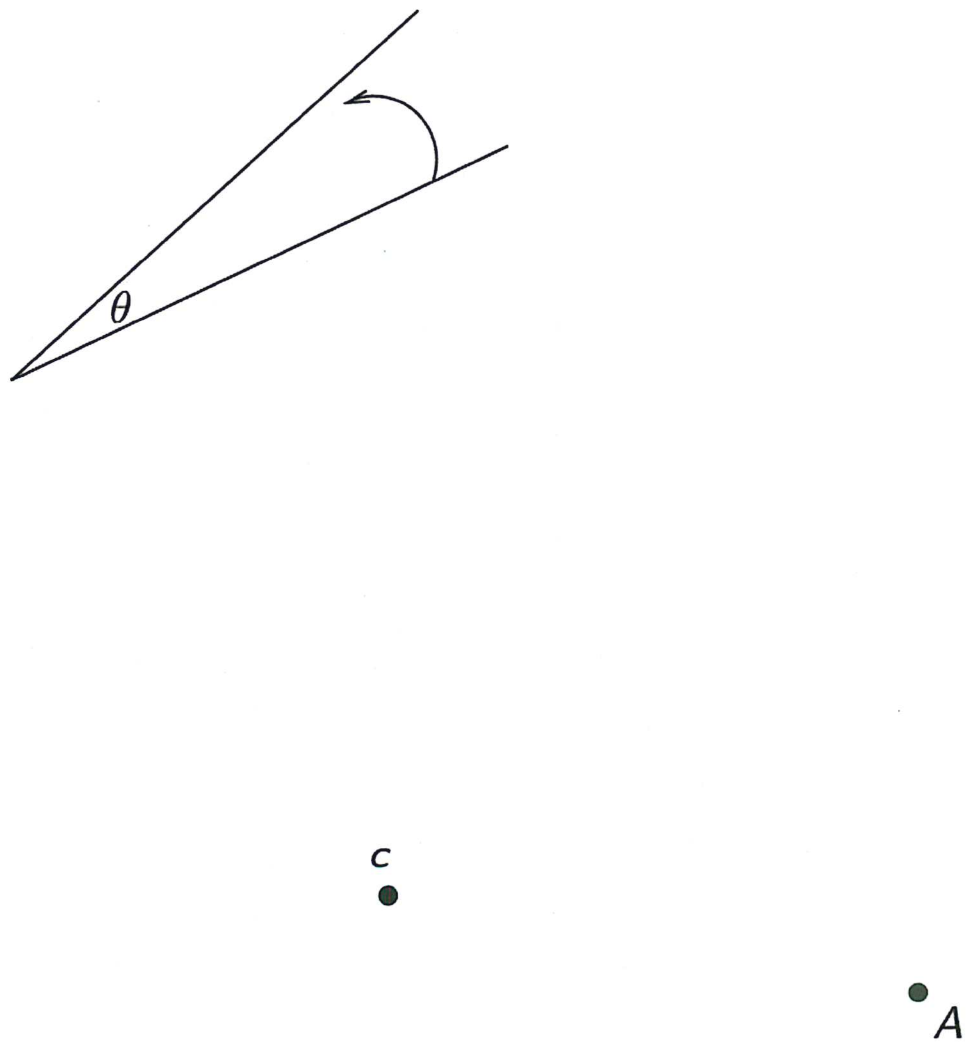
Rotations

A **rotation** is defined by an angle θ and a centre C of a circle, denoted $f = rot(C, \theta)$.



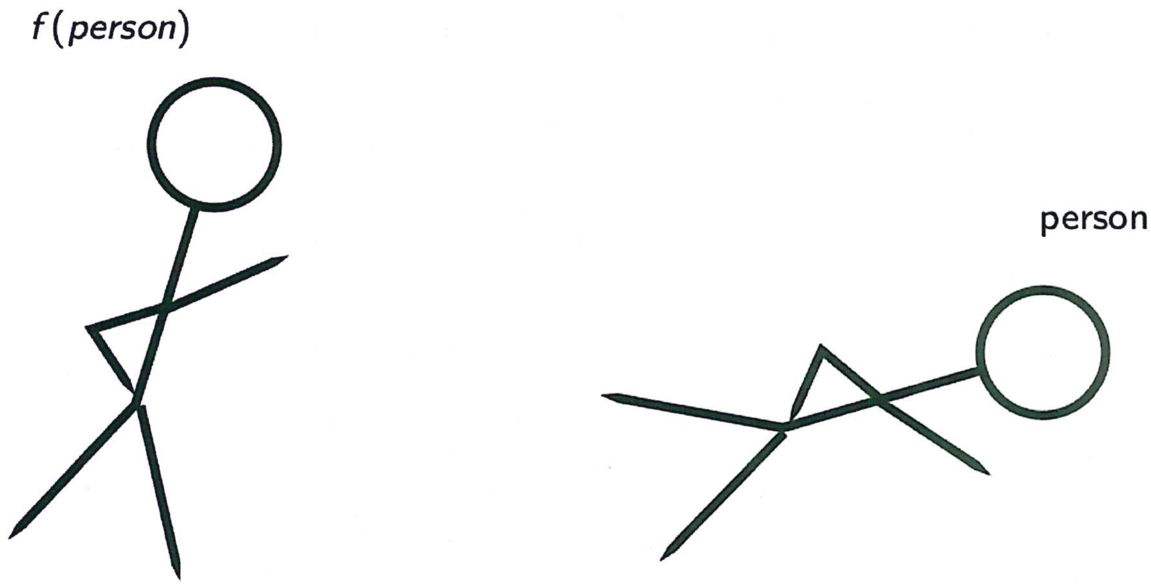
Example

Find the image of A under the symmetry $f = rot(C, \theta)$.

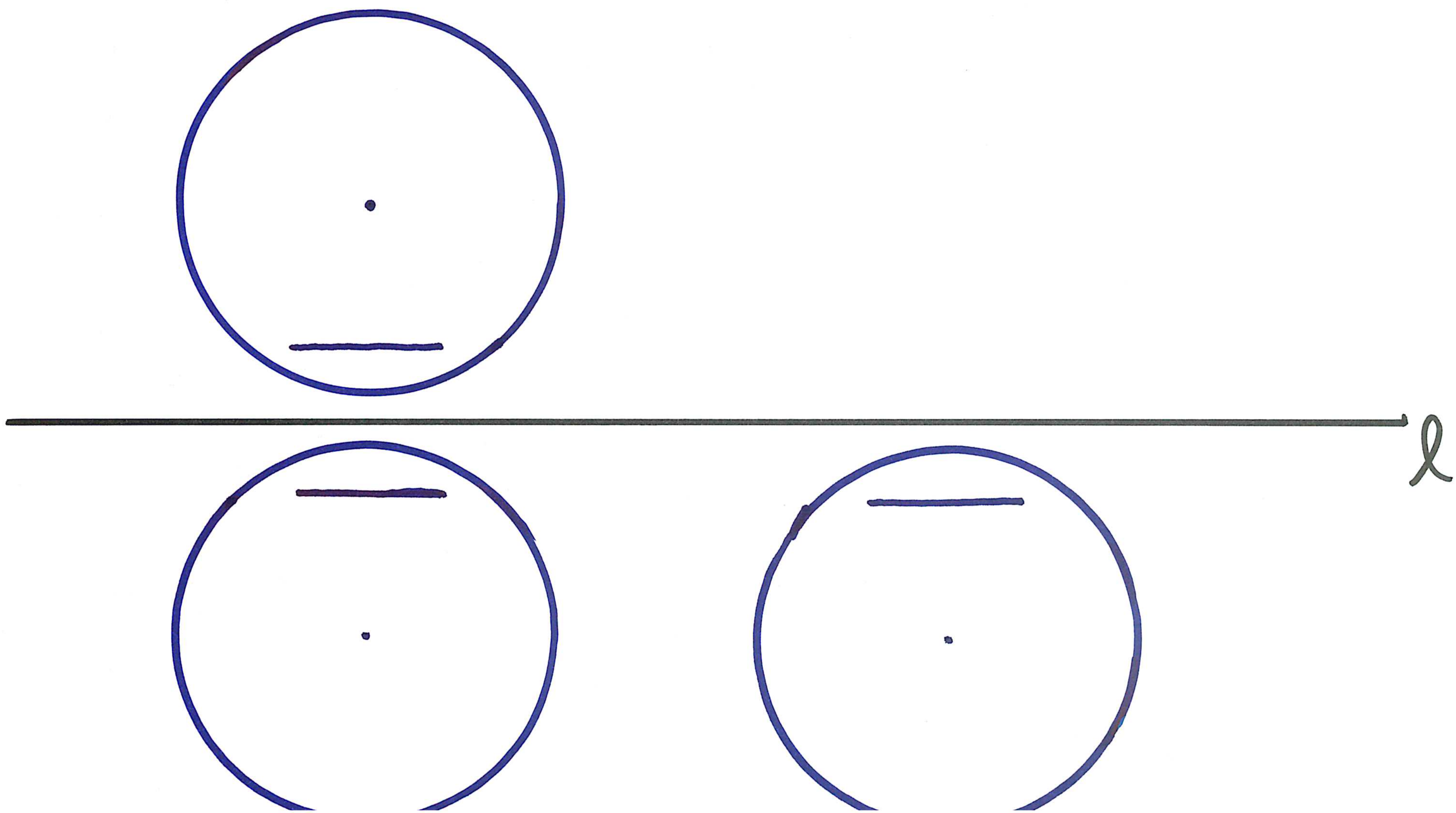


Example

Find the center and angle of the symmetry $f = rot(C, \theta)$.



Example



The Classification Theorem For Plane Symmetries

The composition of two symmetries is also a symmetry!

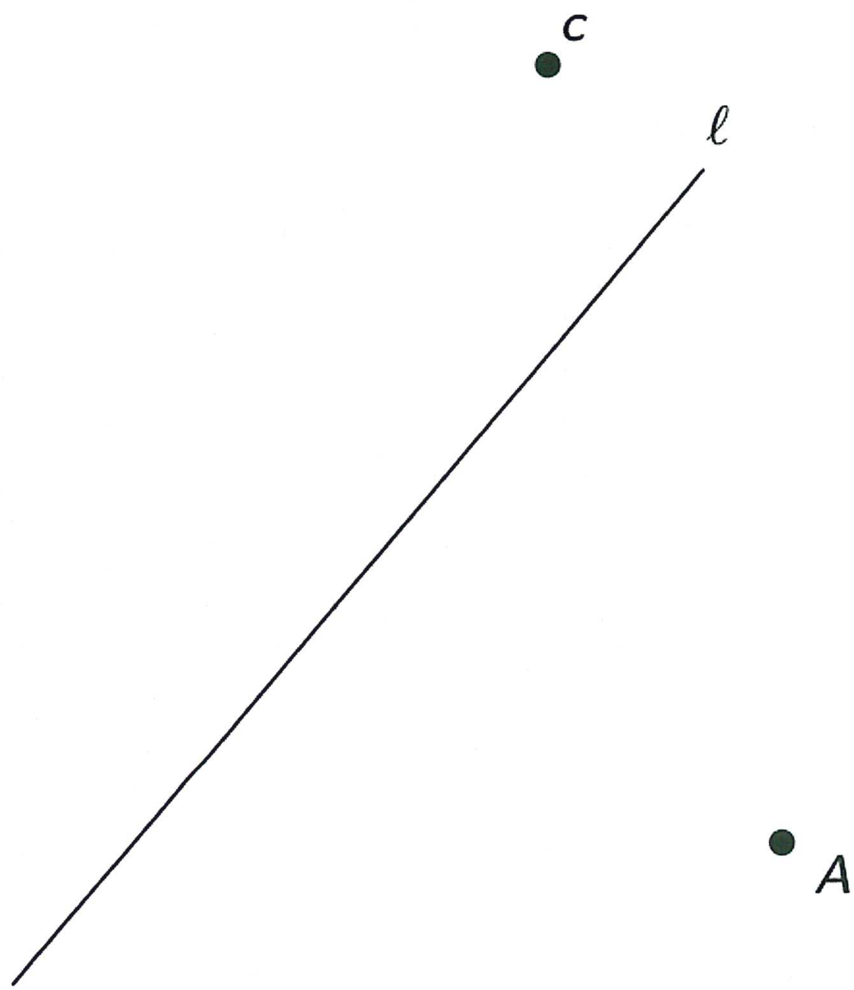
Theorem

Every symmetry of the plane is either:

- *a composition of a translation followed by a rotation; or*
- *a composition of a translation followed by a reflection.*

Example

Find the image of A under the composition of the symmetries $f_1 = refl(\ell)$ followed by $f_2 = rot(C, 60°)$.



Definitions (page 55 & 56 of text)

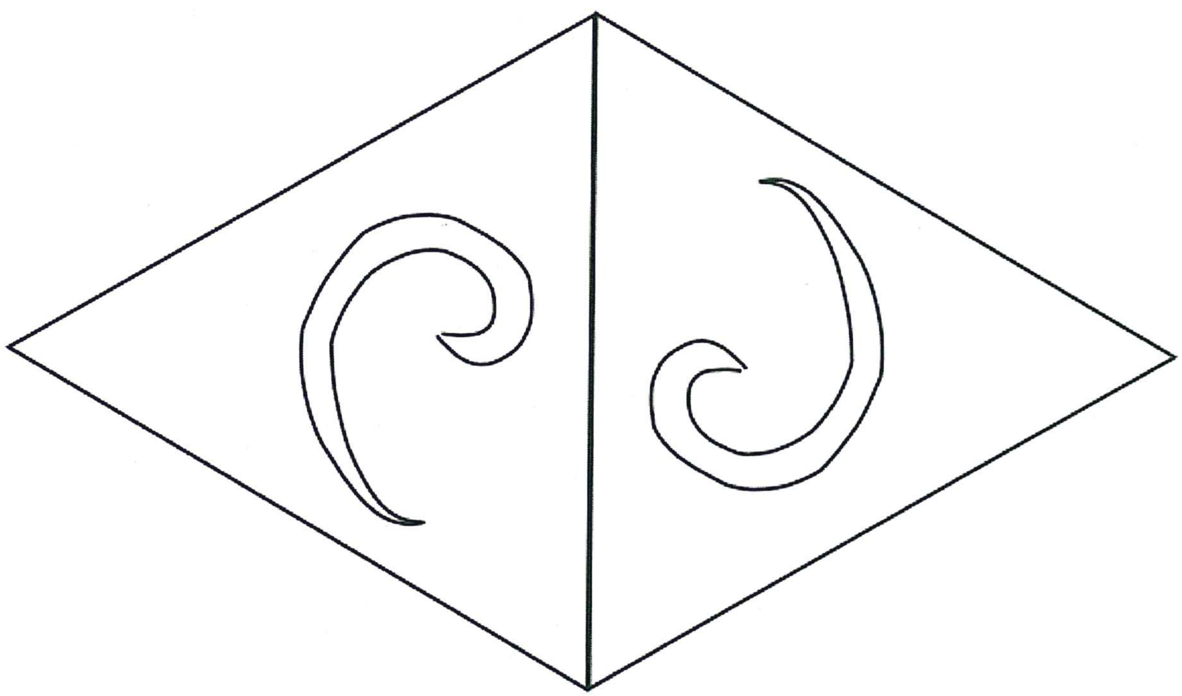
- Given an object O in the plane, a **symmetry of the object** O is a symmetry of the plane that rearranges the points of O within the points of O such that every position in the object is attained by some point following the rearrangement.

  Note: We can think of this as a symmetry under which the object

- The set of all symmetries of an object is called the

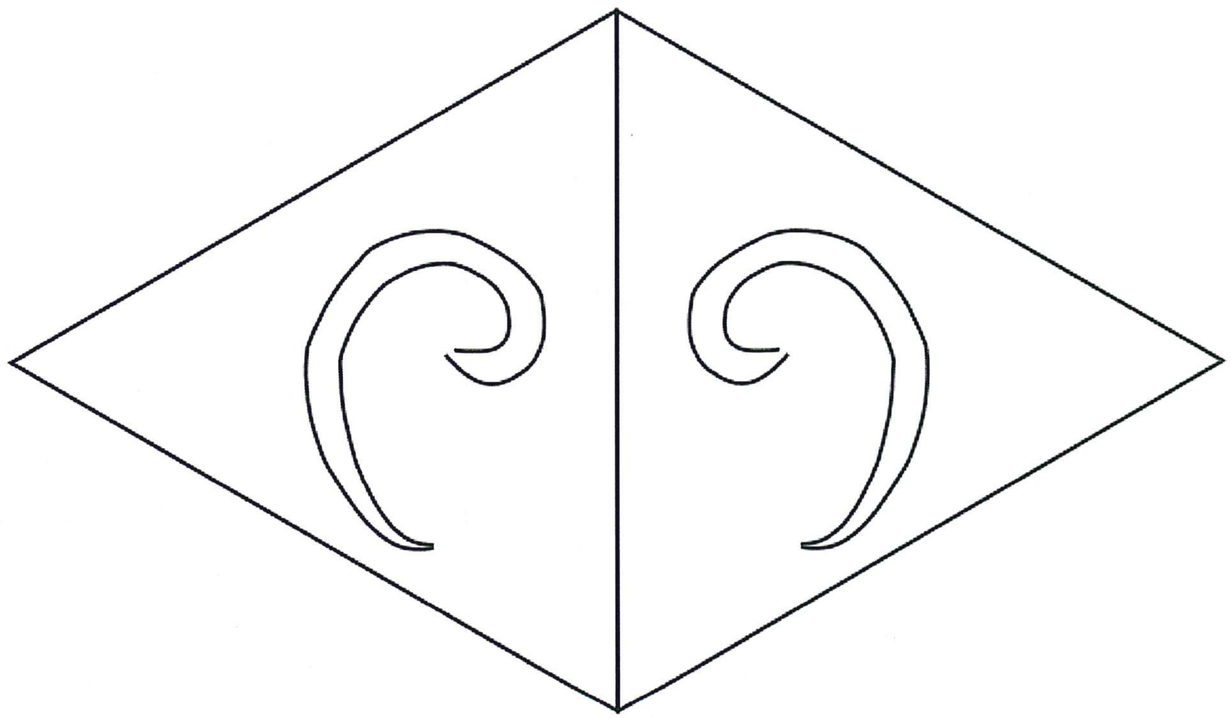
Example

Find the group of symmetries of the following object.



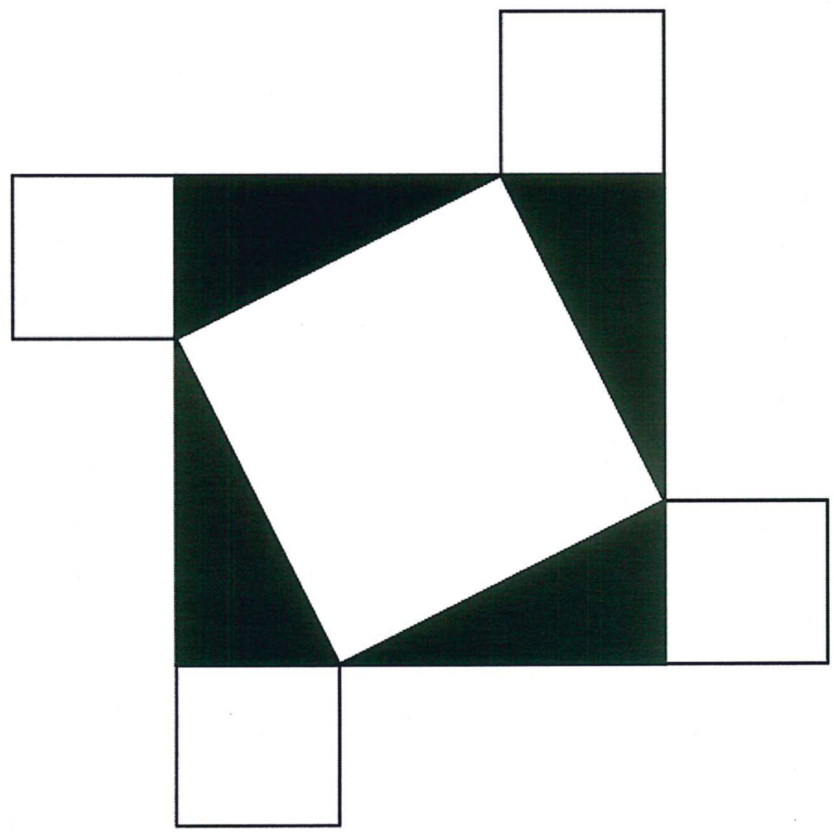
Example

Find the group of symmetries of the following object.



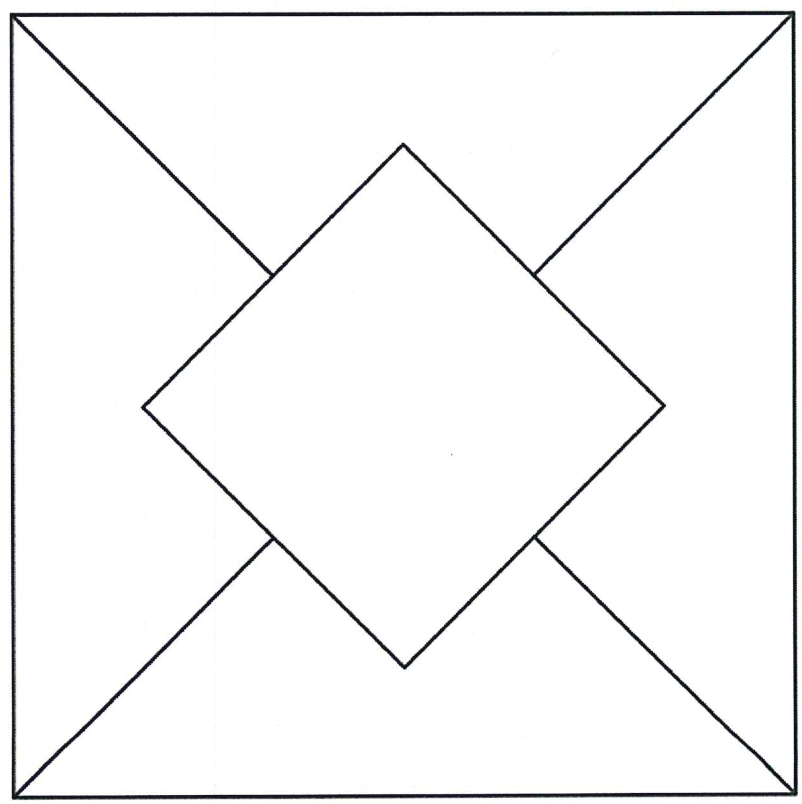
Example

Find the group of symmetries of the following object.



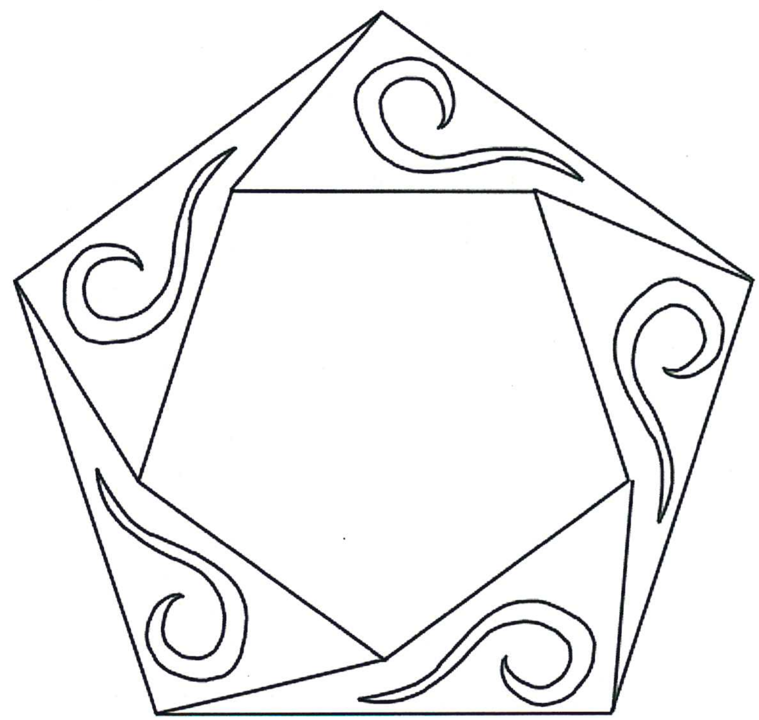
Example

Find the group of symmetries of the following object.



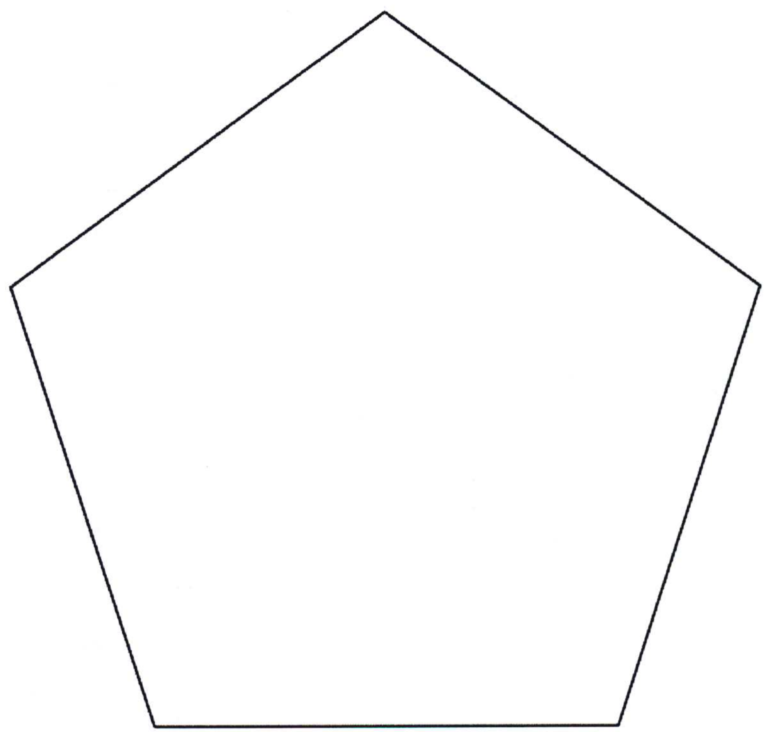
Example

Find the group of symmetries of the following object.



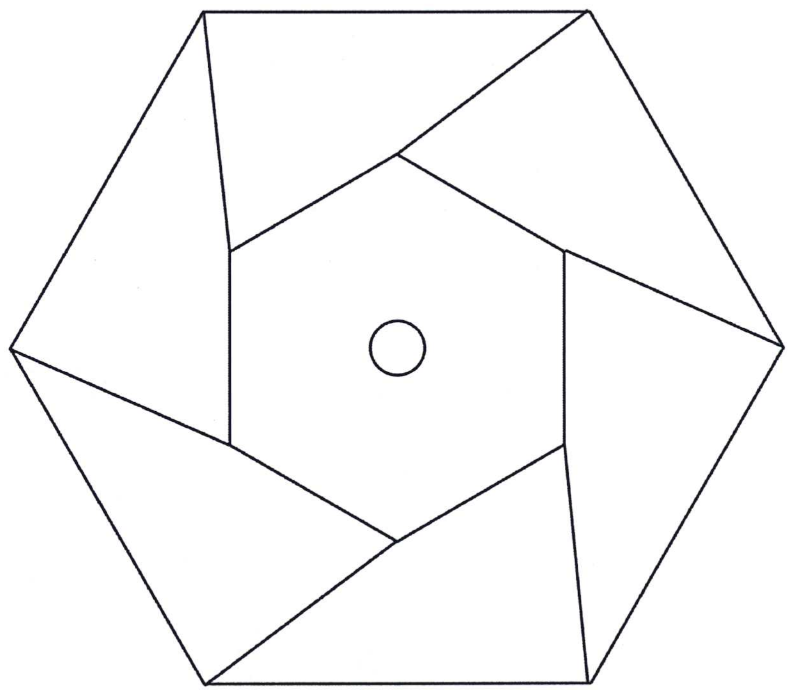
Example

Find the group of symmetries of the following object.



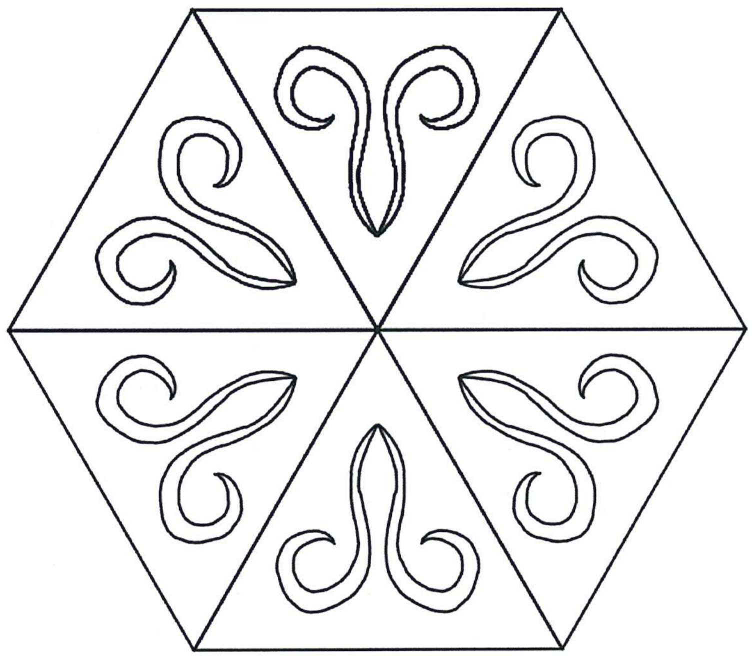
Example

Find the group of symmetries of the following object.



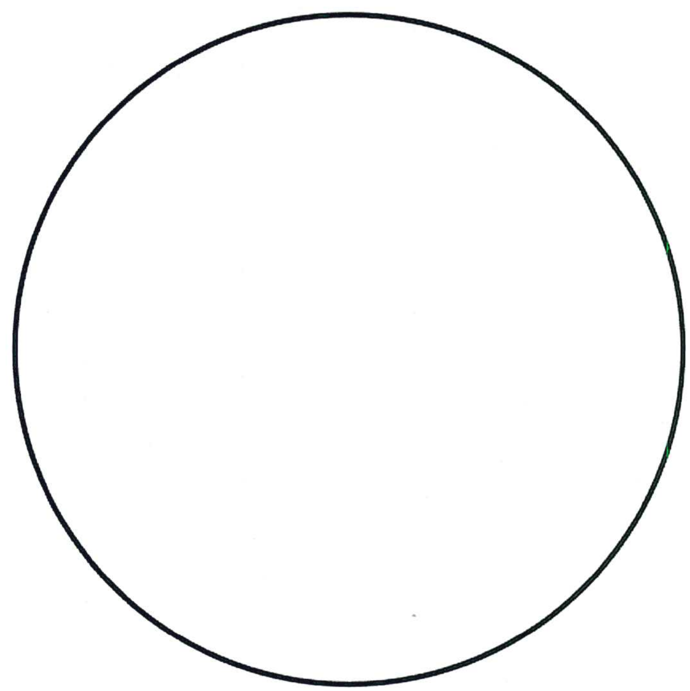
Example

Find the group of symmetries of the following object.



Example

Find the group of symmetries of the following object.

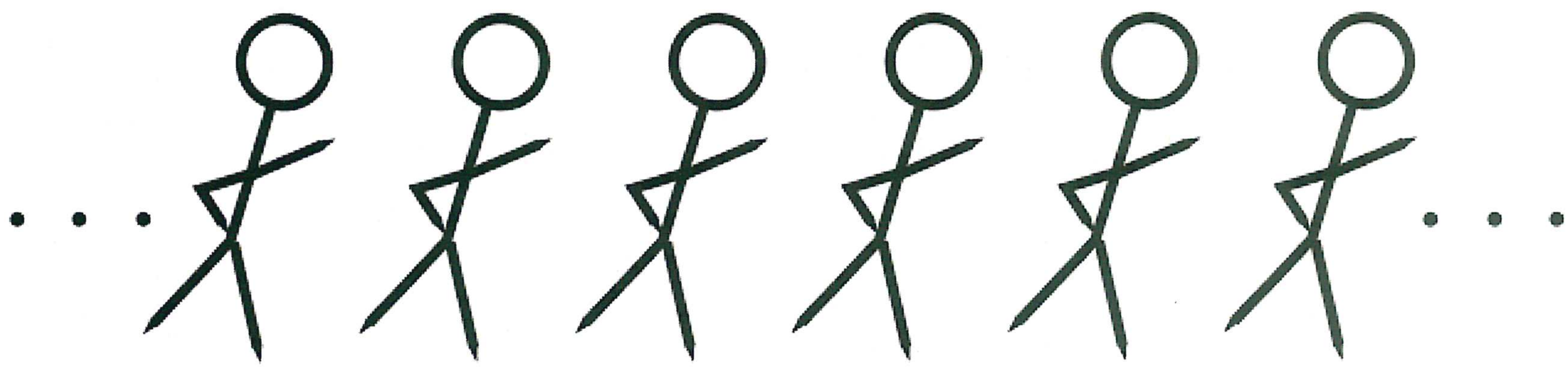


Definition (page 61 of text)

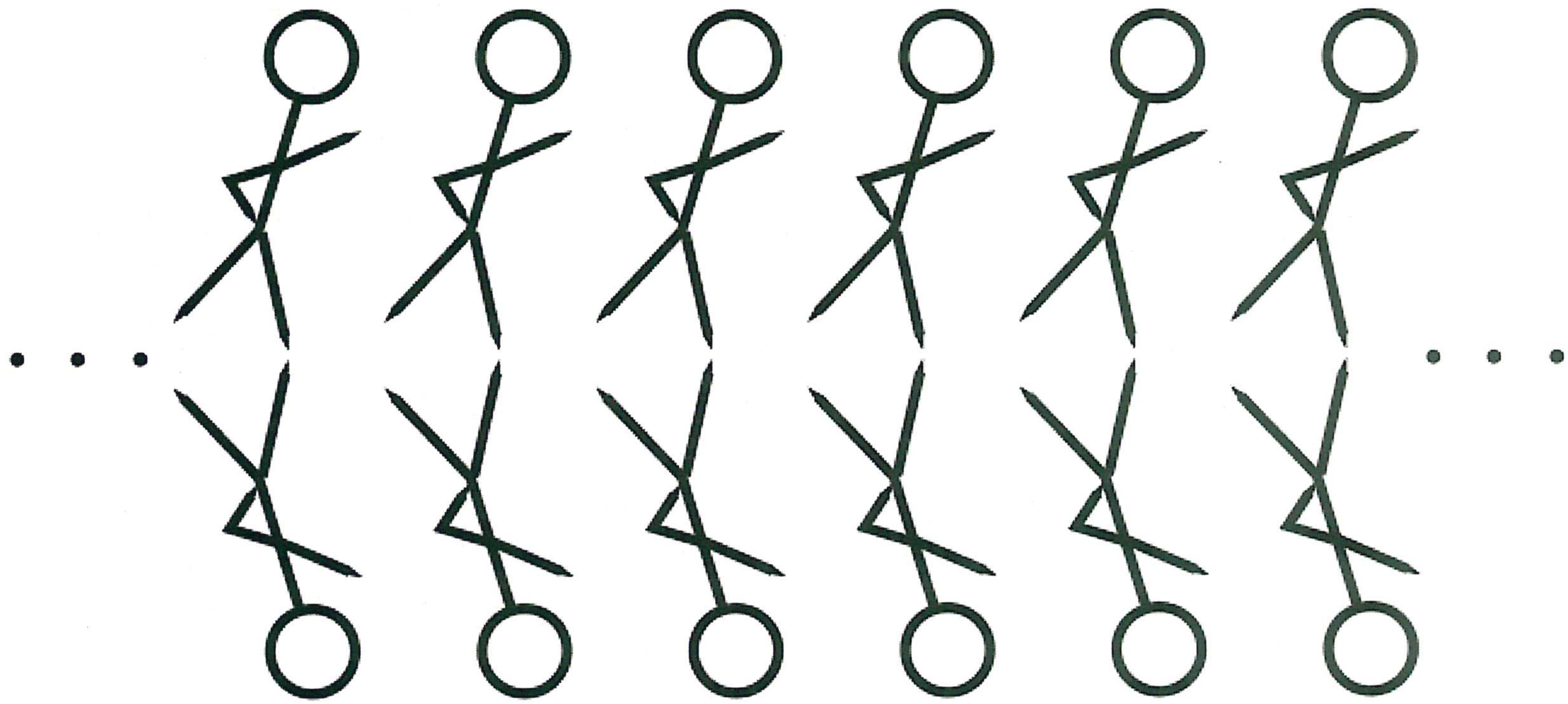
- Objects in the plane that have translation symmetries such that *the vectors of the translation symmetries are all integer multiples of *one* fixed vector* are called

- The groups of symmetries are called

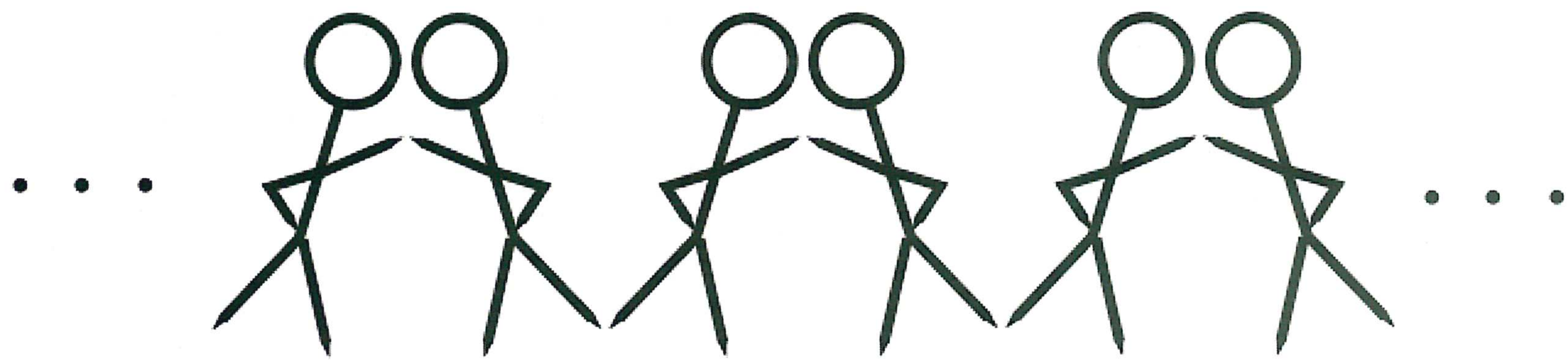
Frieze Patterns - Example



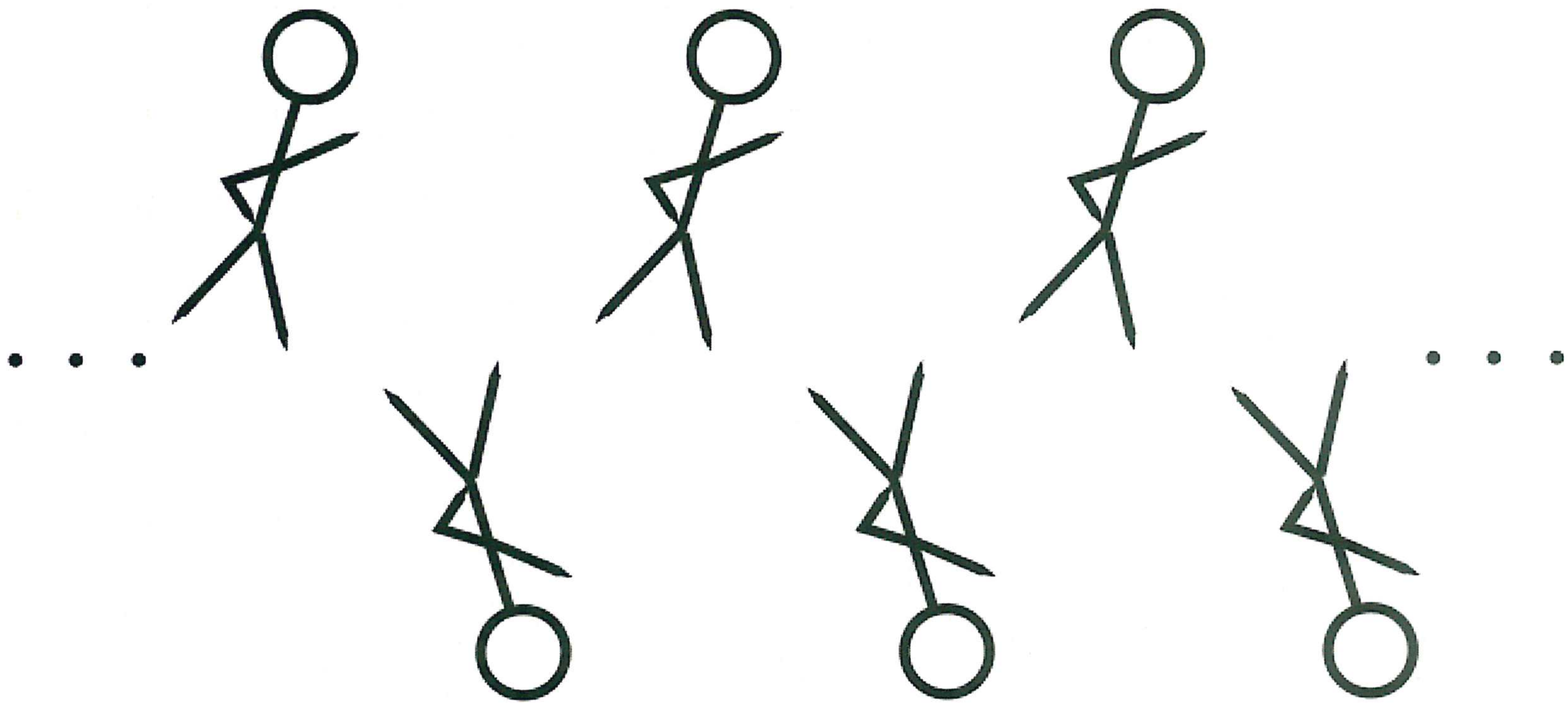
Frieze Patterns - Example



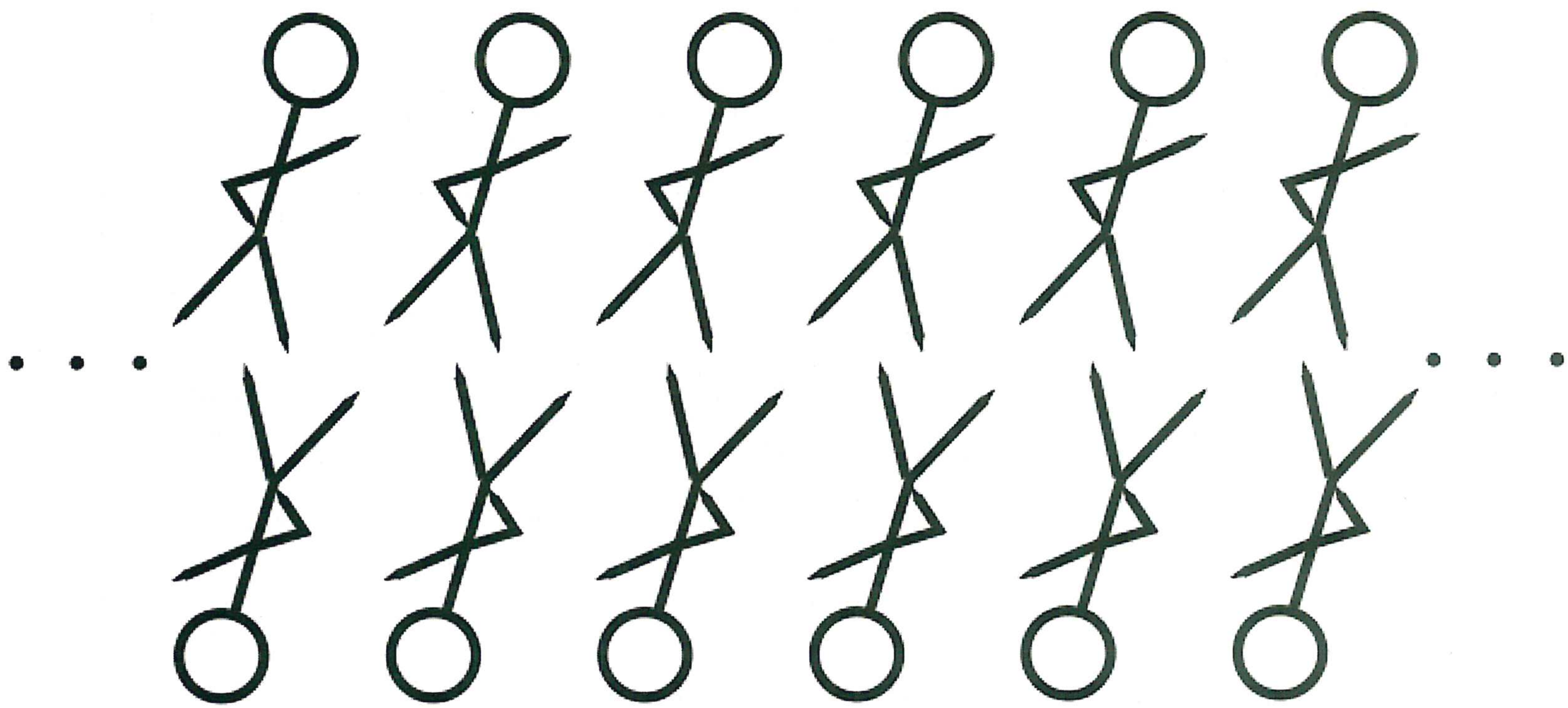
Frieze Patterns - Example



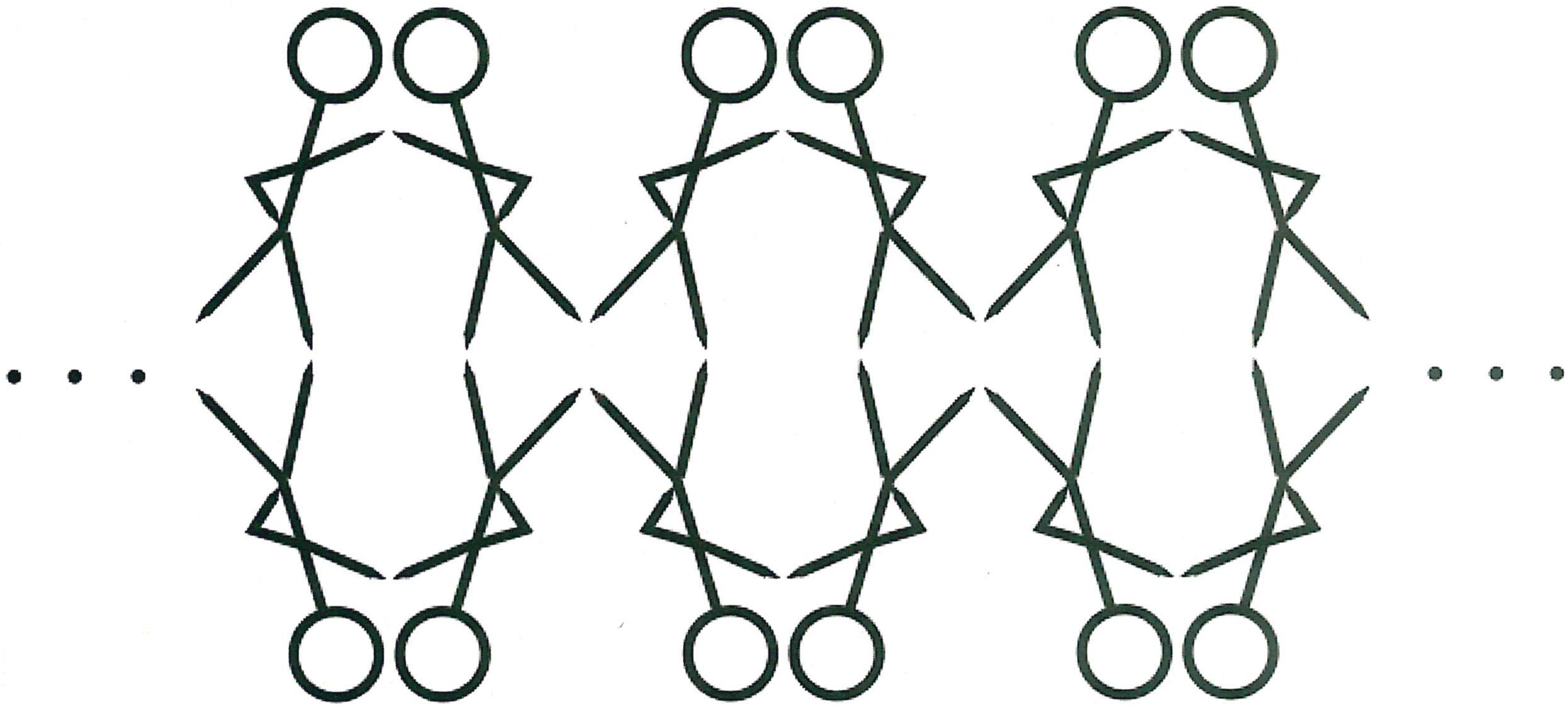
Frieze Patterns - Example



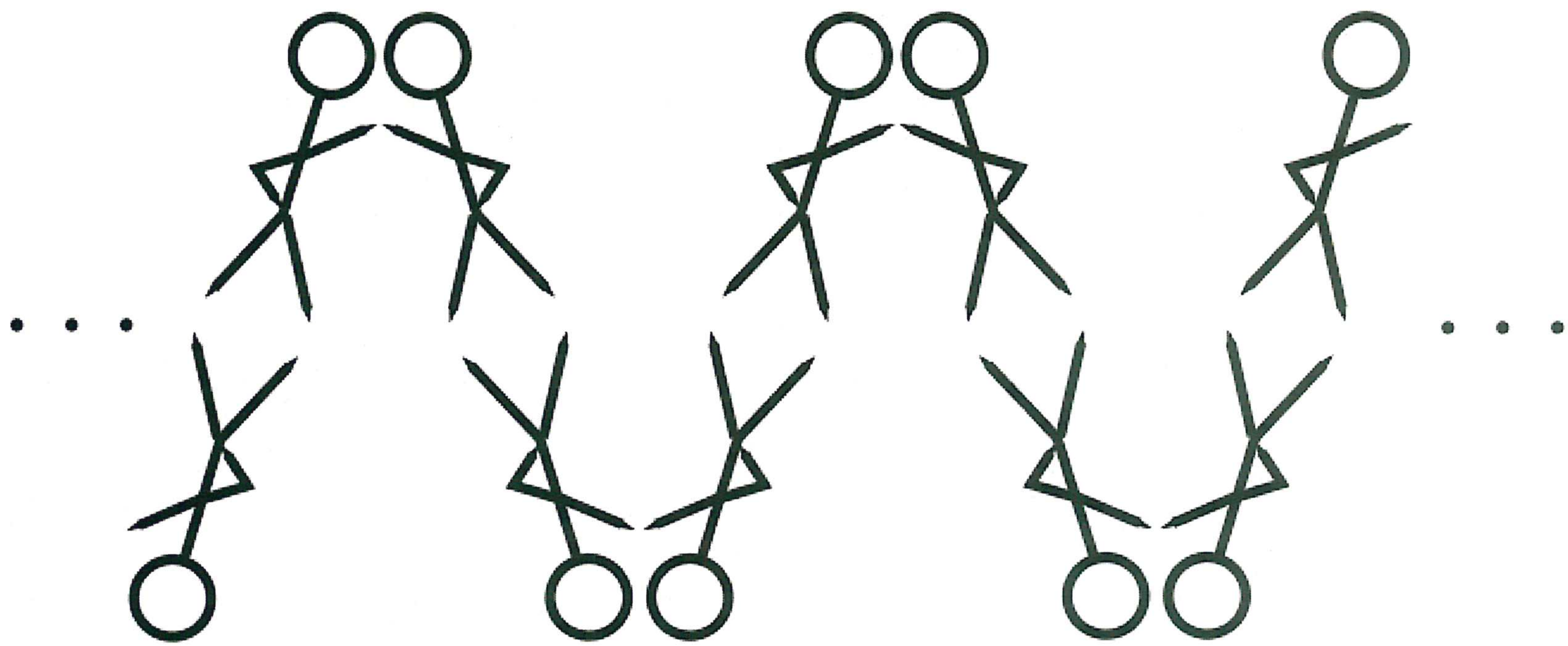
Frieze Patterns - Example



Frieze Patterns - Example



Frieze Patterns - Example



The Classification Theorem For Friezes

Theorem

The seven frieze groups $F_1, F_2, F_3, F_4, F_5, F_6$ and F_7 are the **only** *frieze groups.*

Definitions (page 73 of text)

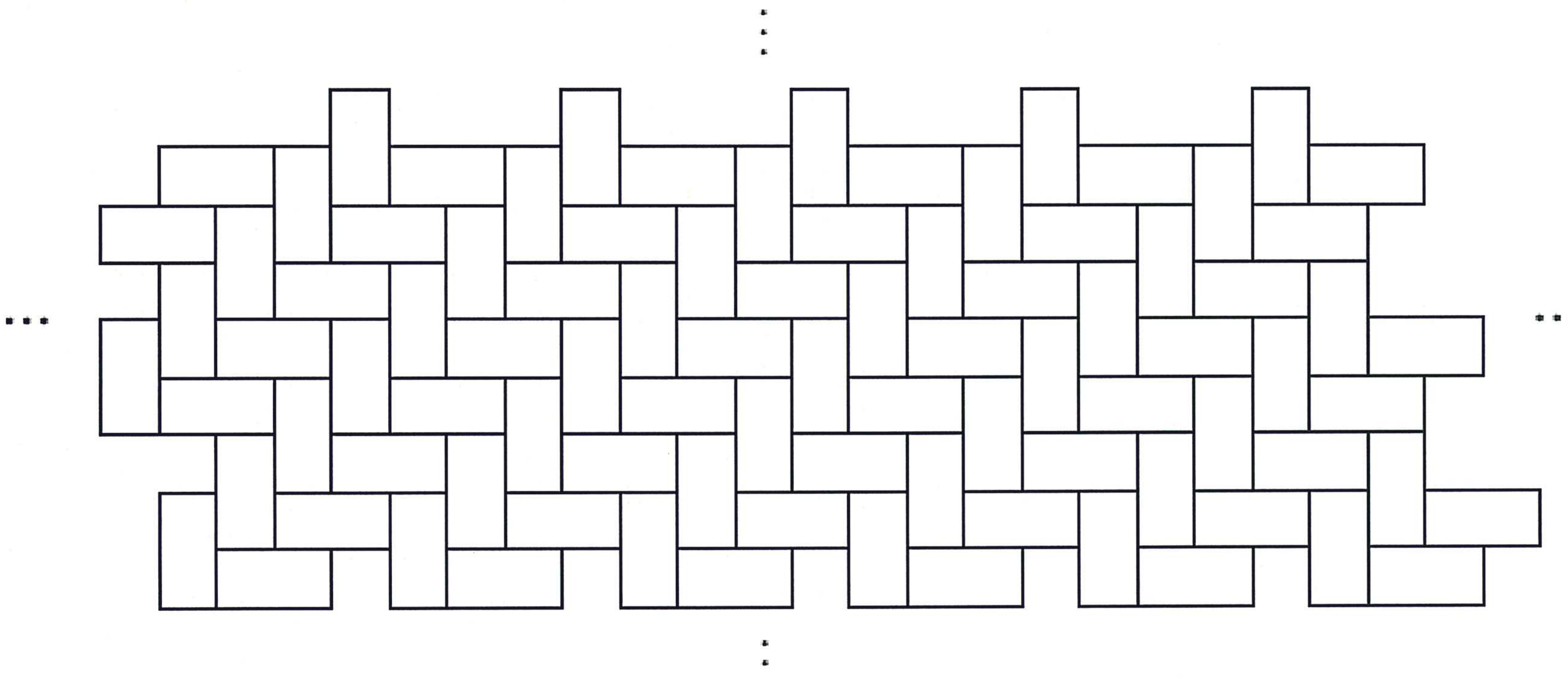
- An object in the plane is a **wallpaper design** if:

  (a) There are two non-parallel vectors such that every translation that can be obtained by composing a translation along an integer multiple of the first vector followed by a translation along an integer multiple of the second vector is a symmetry of the object.

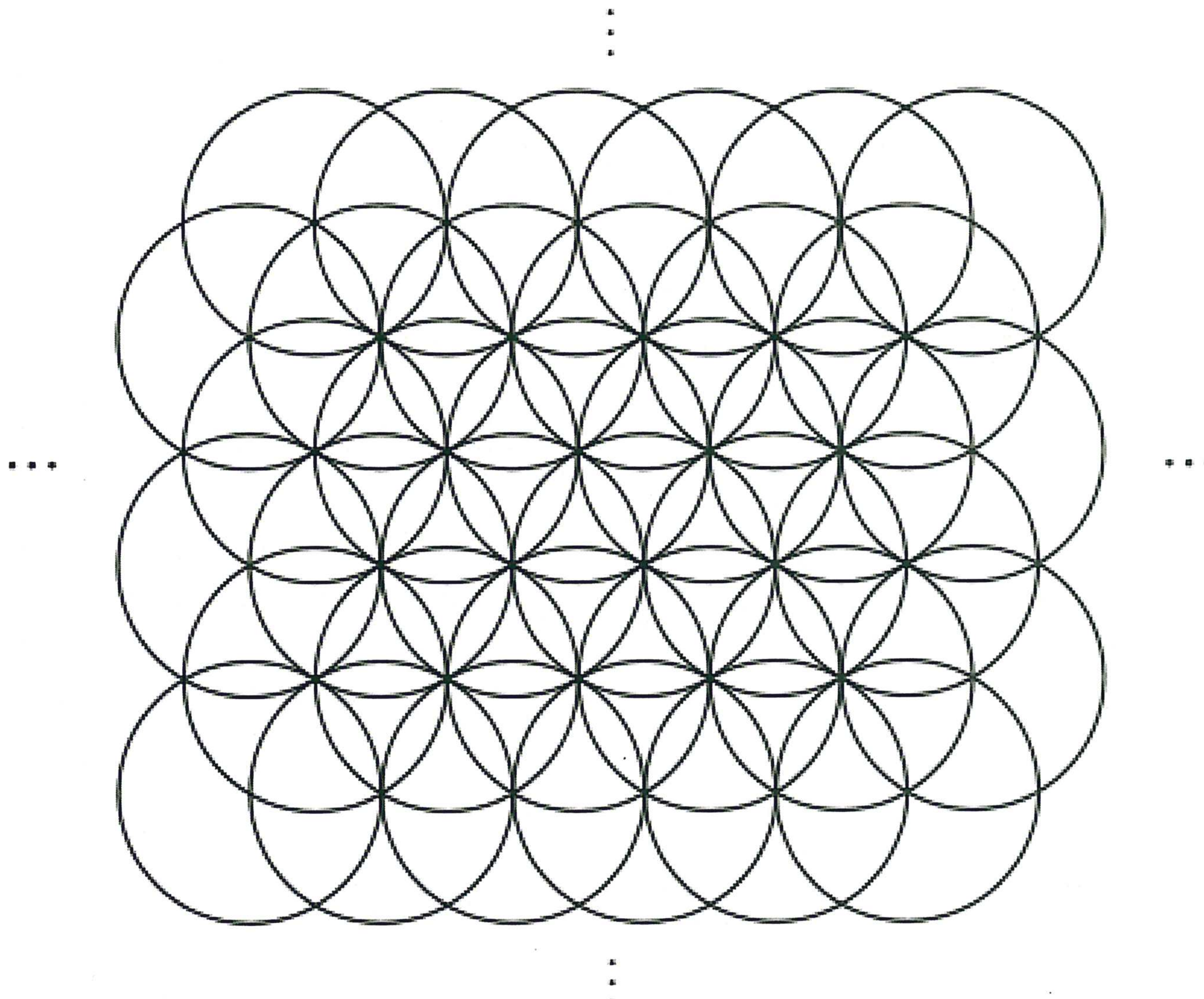
  (b) Every translation symmetry of the object must be of the kind specified in (a).

- The groups of symmetries of wallpaper designs are called **wallpaper groups**.

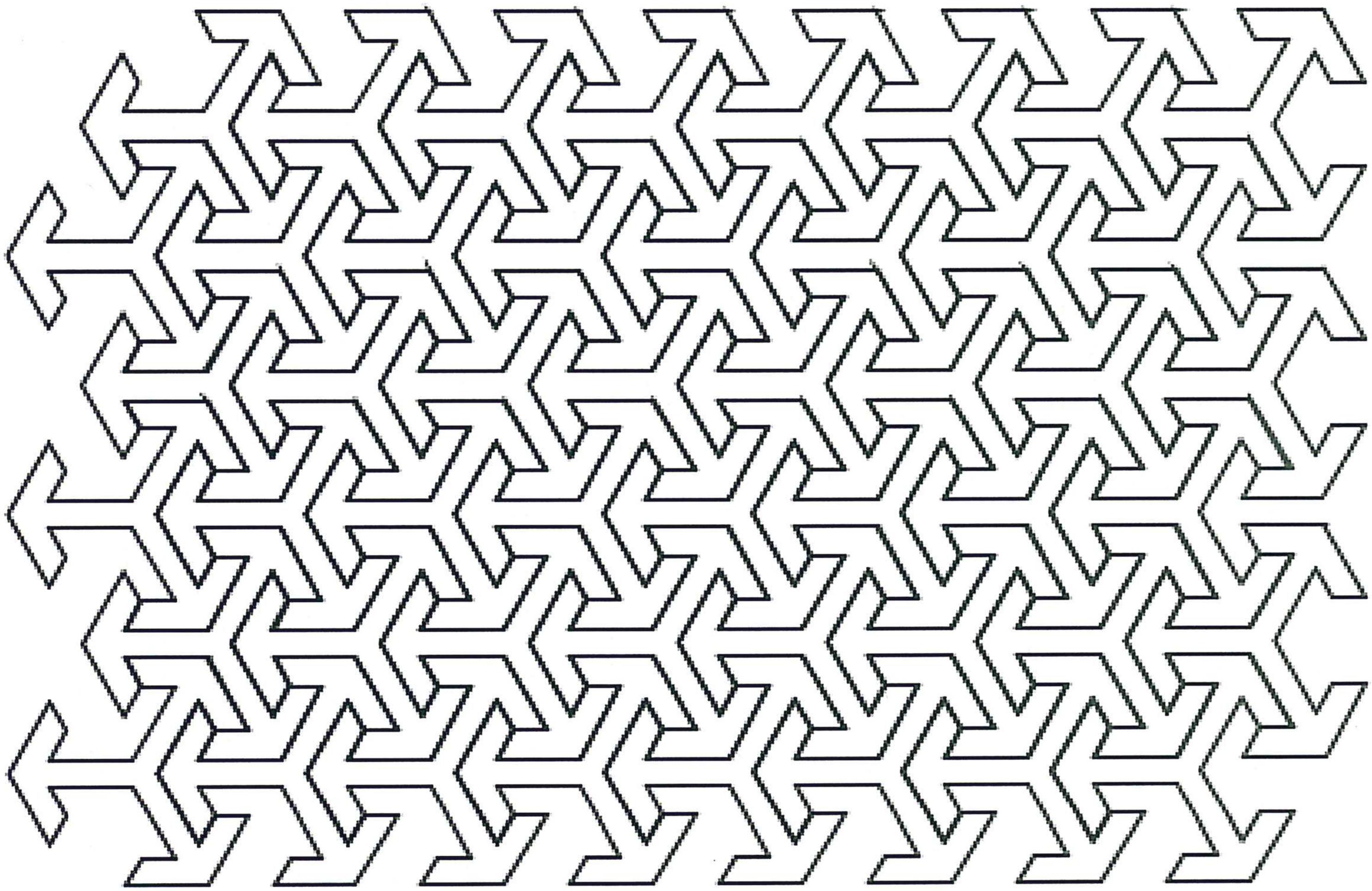
Wallpapers - Example



Wallpapers - Example



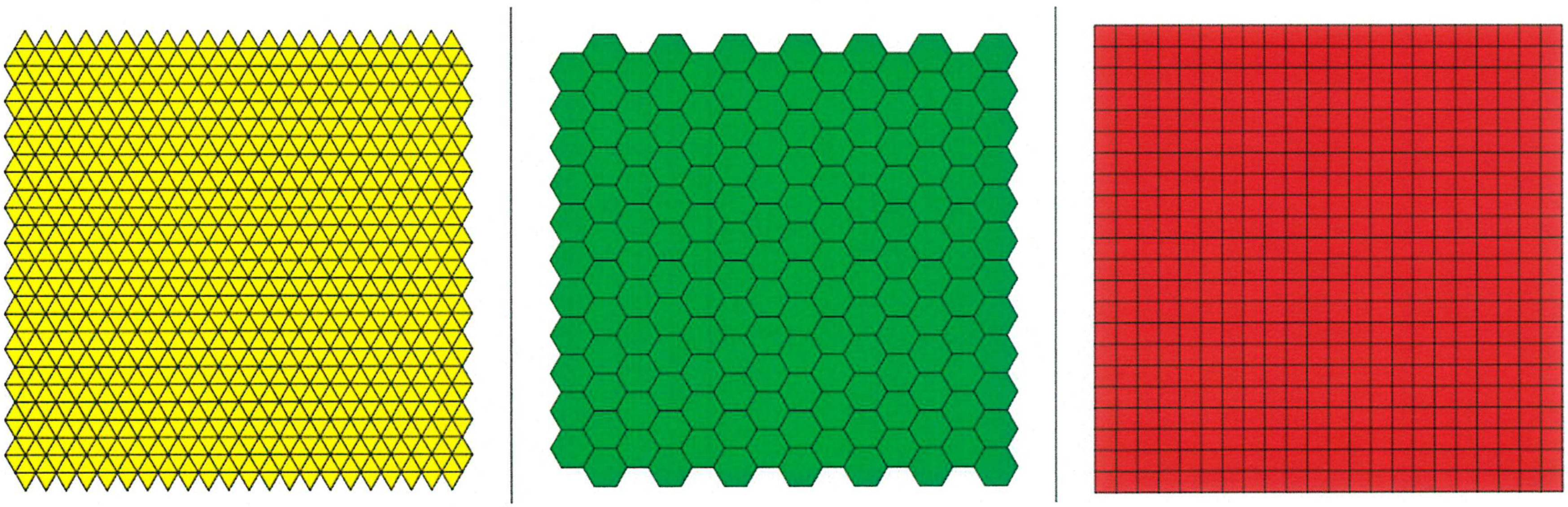
Wallpapers - Example



Definition (page 74 of text)

Any arrangement of objects on a plane in such a way that all of the plane is covered and such that any two tiles either share a common corner, intersect along a pair of their edges, or do not intersect at all, is called a **tiling** of the plane. The objects used to cover the plane are called **tiles**.

Regular Tilings Of The Plane



Regular Tilings Of The Plane (pages 77–79 of text)

A tiling of the plane is **regular** if:

- all the tiles used are regular polygons (equilateral triangles, squares, regular pentagons, etc.);
- every two adjacent polygons have either a common point or a common edge;
- if the polygons around every common point are of the same type.

Monohedral Tilings

Theorem

There are exactly 3 monohedral regular tilings of the plane.

Archimedean Tilings

Useful Reference

- https://en.wikipedia.org/wiki/Euclidean_tilings_by_convex_regular_polygons

QUESTIONS???