Fractals: Self-Similarity

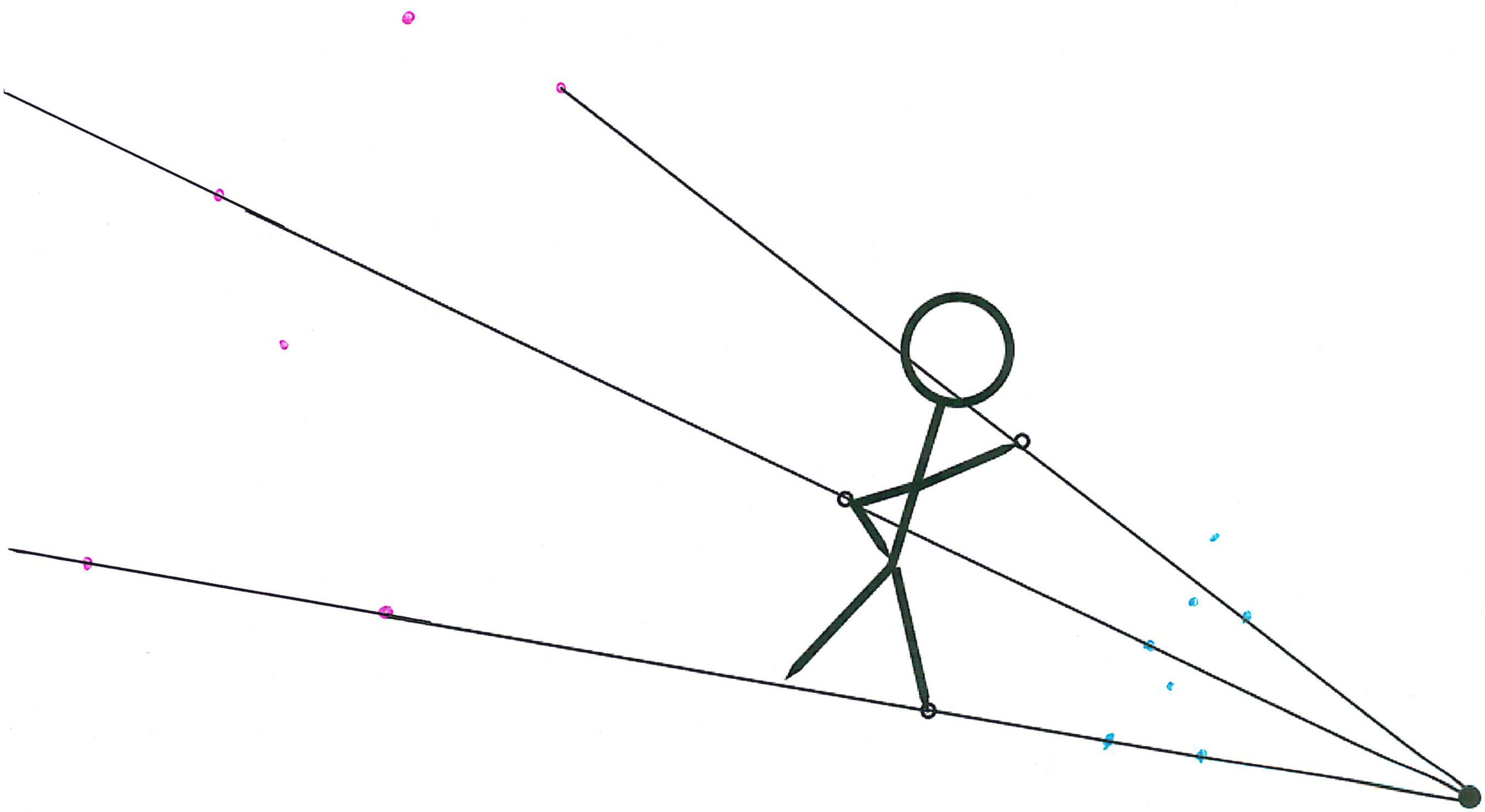
Definition (page 92 of text)

A plane transformation f is called a **similarity** if there exists a positive number α such that for every two points A and B on the plane, we have

Note: If $\alpha = 1$ then

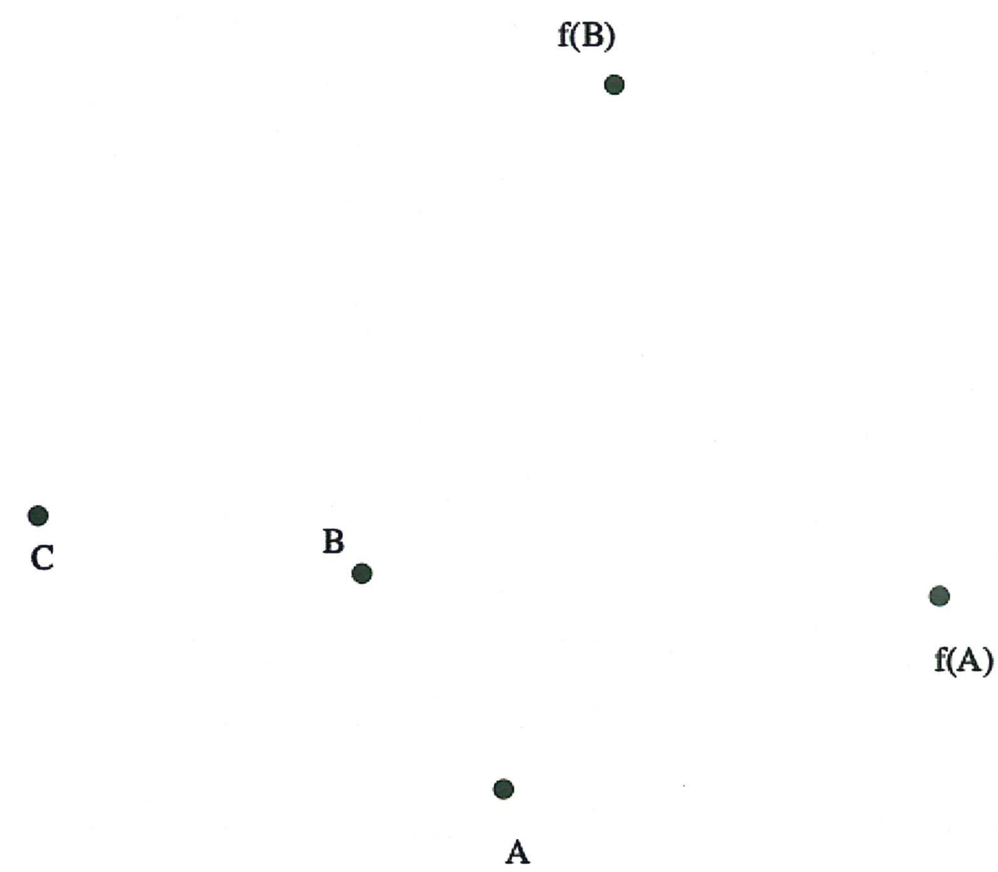
- The number α is called the

Example: Central Similarities (Dilations)



Example

Find: the center of the central similarity f; the image of C under f.



Spiral Similarity

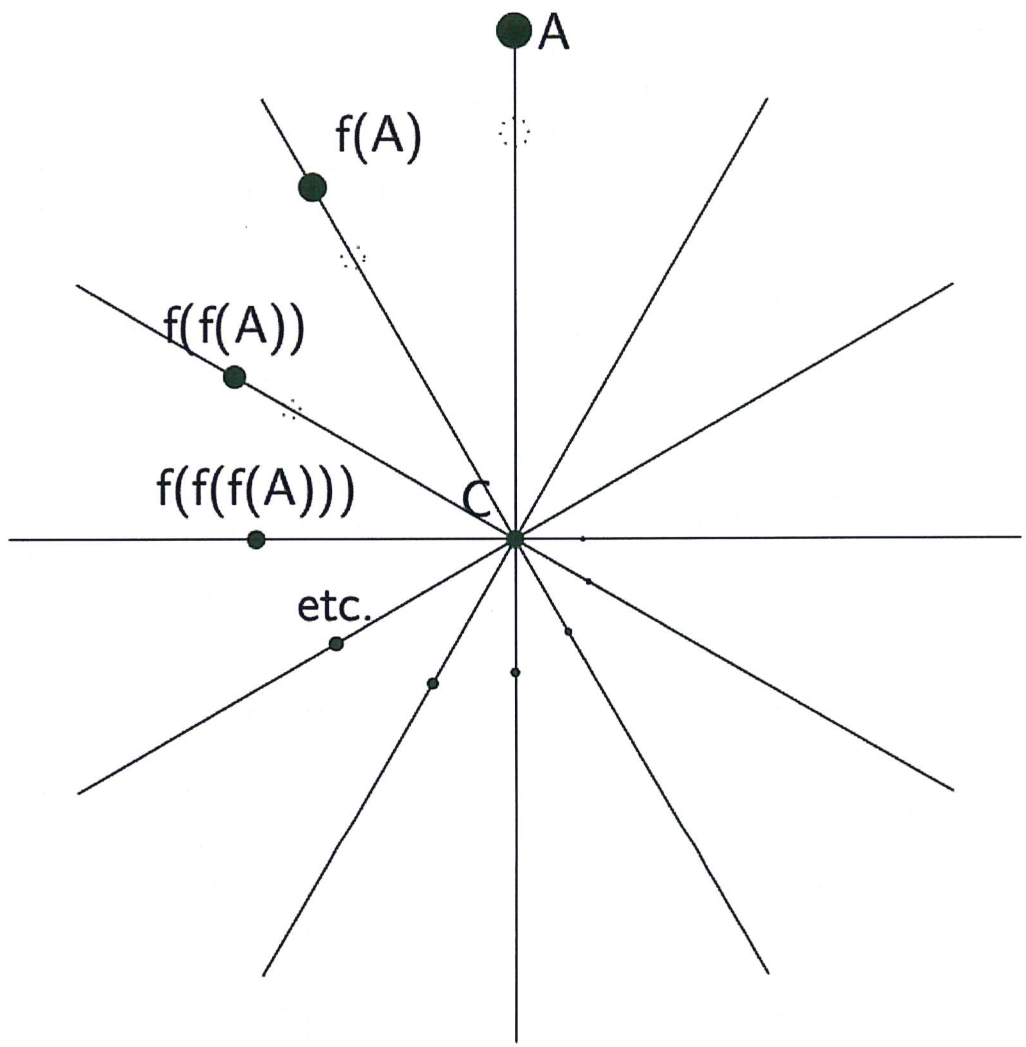
Let f be a composition of a

Note: f is a similarity.

- If the center is the same point for both the rotation and central similarity, then f is called a **spiral similarity**.

Example

- $\alpha = 0.8, \theta = 30°$, center = C



Dilative Reflection

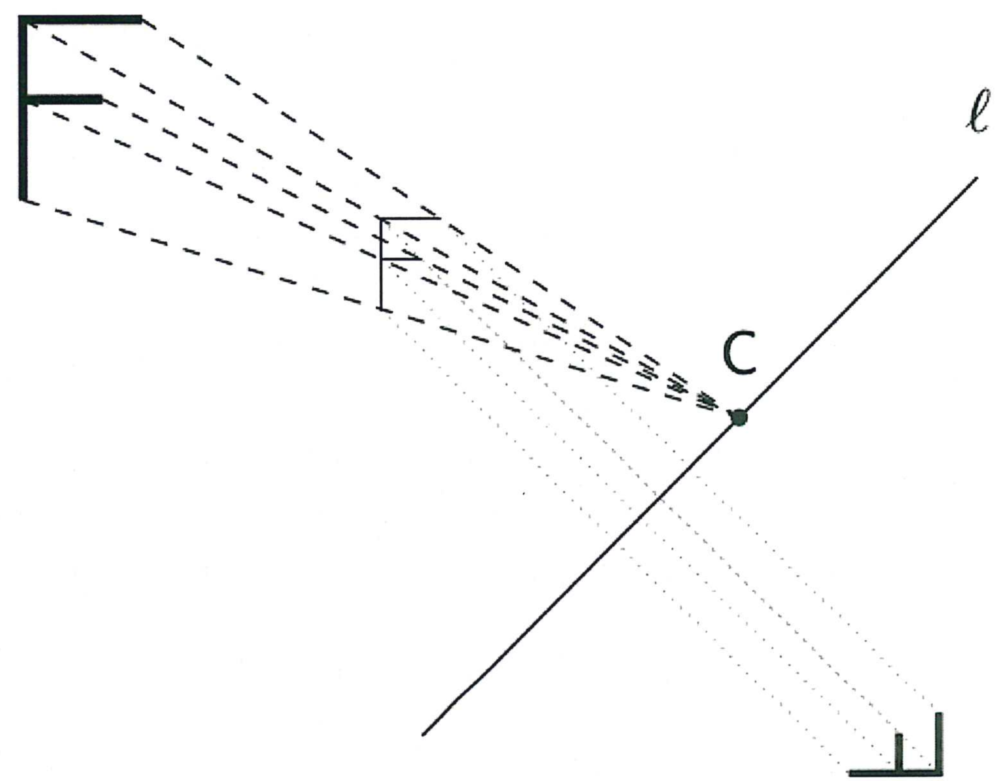
Let f be a composition of a

Note: f is a similarity.

- If the center of the central similarity is on the line of reflection, then f is called a **dilative reflection**.

Example

- $\alpha = 0.5$, center $= C$, line $= \ell$



Classification For Similarities

Theorem

Every similarity is a symmetry, a spiral similarity, or a dilative reflection.

Similar Objects

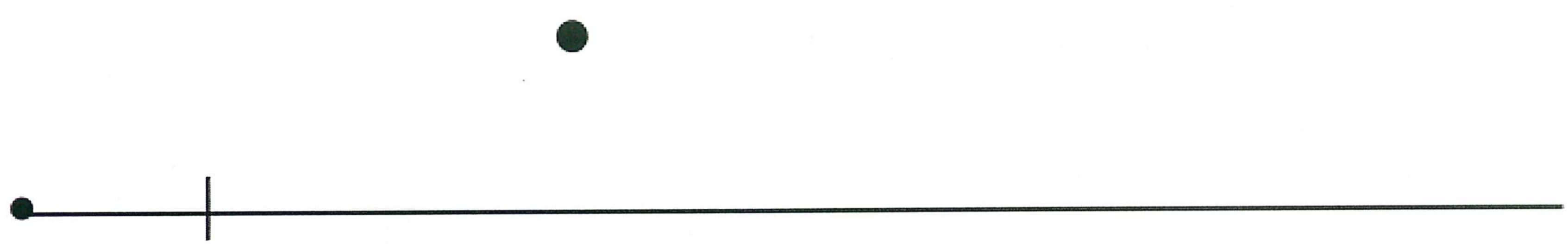
Two objects are **similar** if they have the same shape, regardless of orientation.

Example: Similar Always? Never? Sometimes?

- two rectangles having the same area
- two Golden triangles
- two isosceles triangles with different heights
- two isosceles triangles with the same height
- two Golden obtuse triangles having different heights
- two circles of different diameters
- two pentagons
- a square and a Golden Rectangle

Squaring Transformation

Square the distance, double the angle.



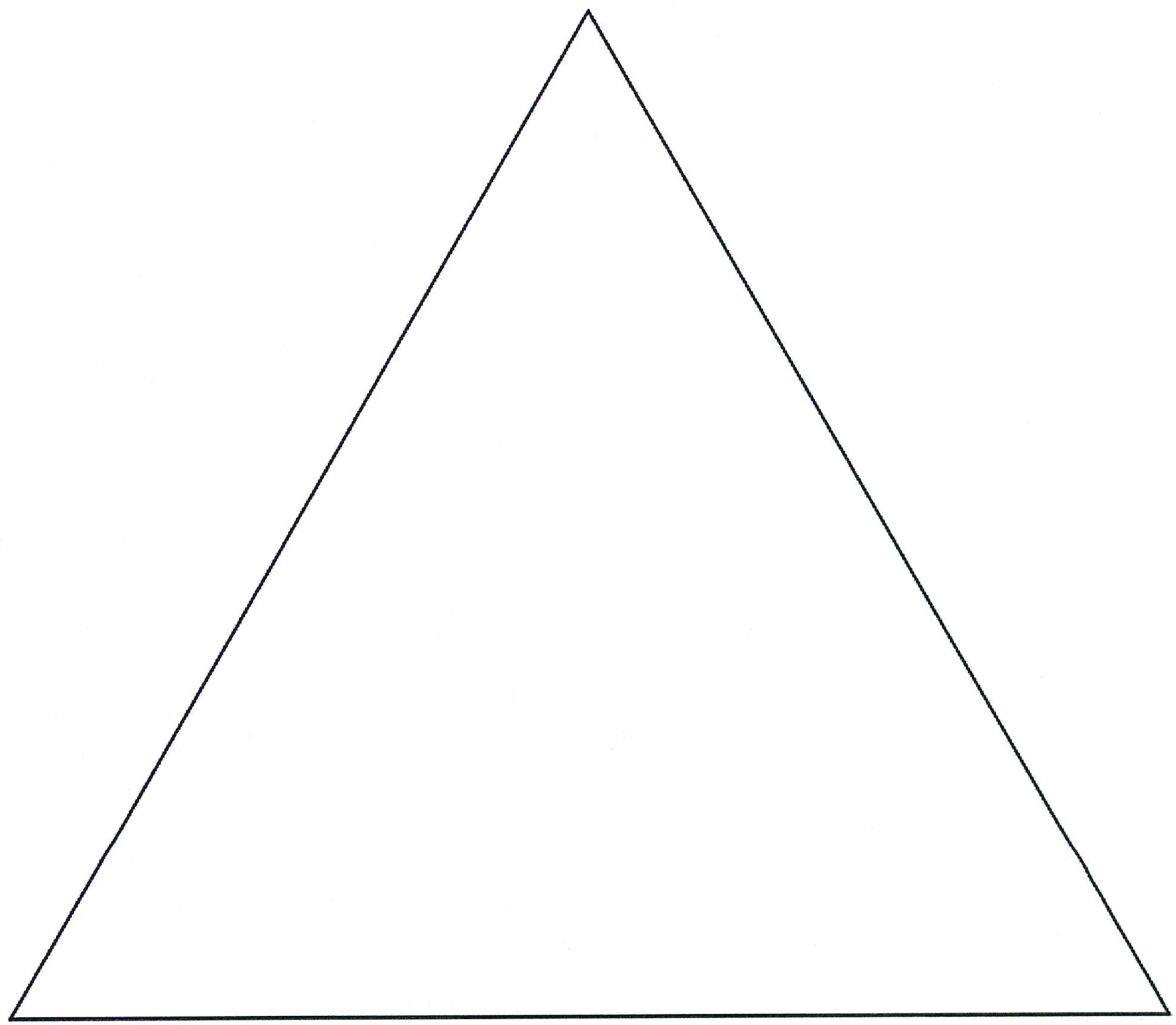
Definition (page 110 of text)

A **fractal** is an object O that possesses the property of *proper self-similarity*.

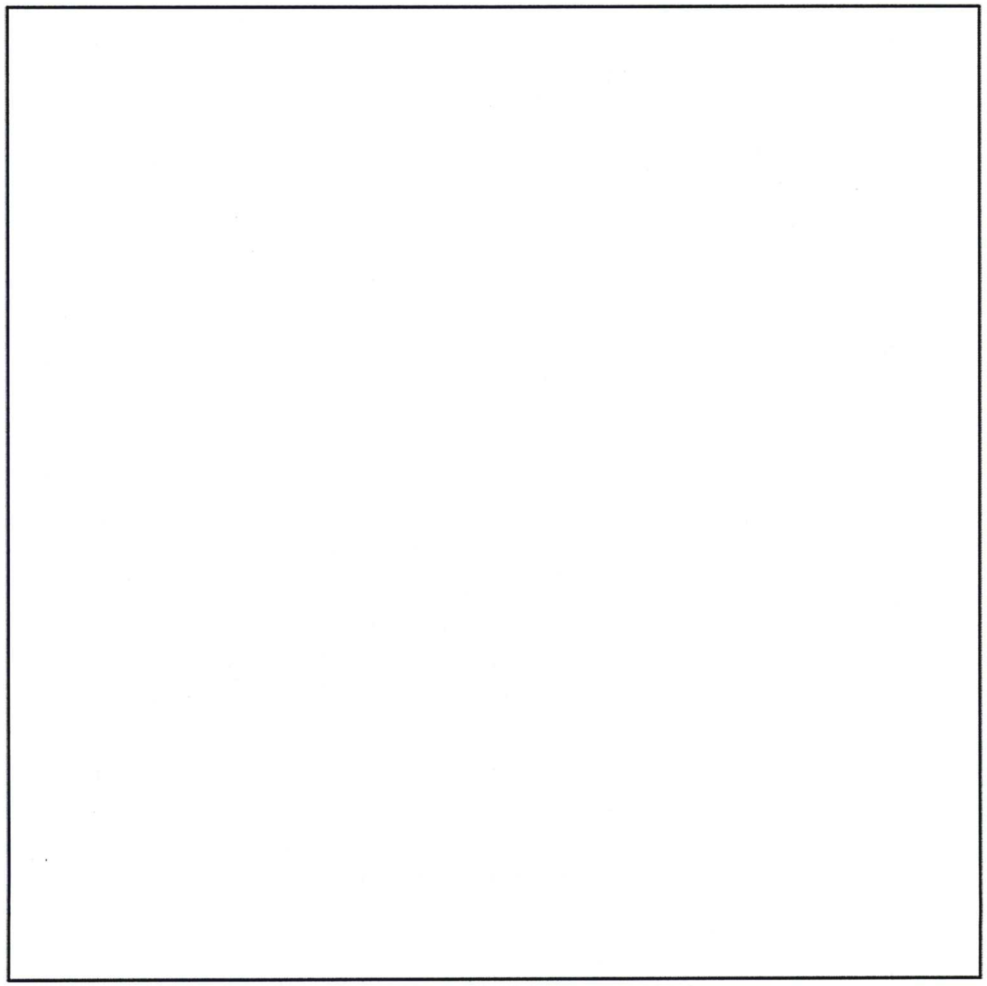
- This means that there is a part of O, say A_1, which is
- That is, there is a similarity that sends a part A_1 of O *onto* a proper part A_2 of A_1.

Note: The similarity cannot be a

Example: Sierpinski Triangle

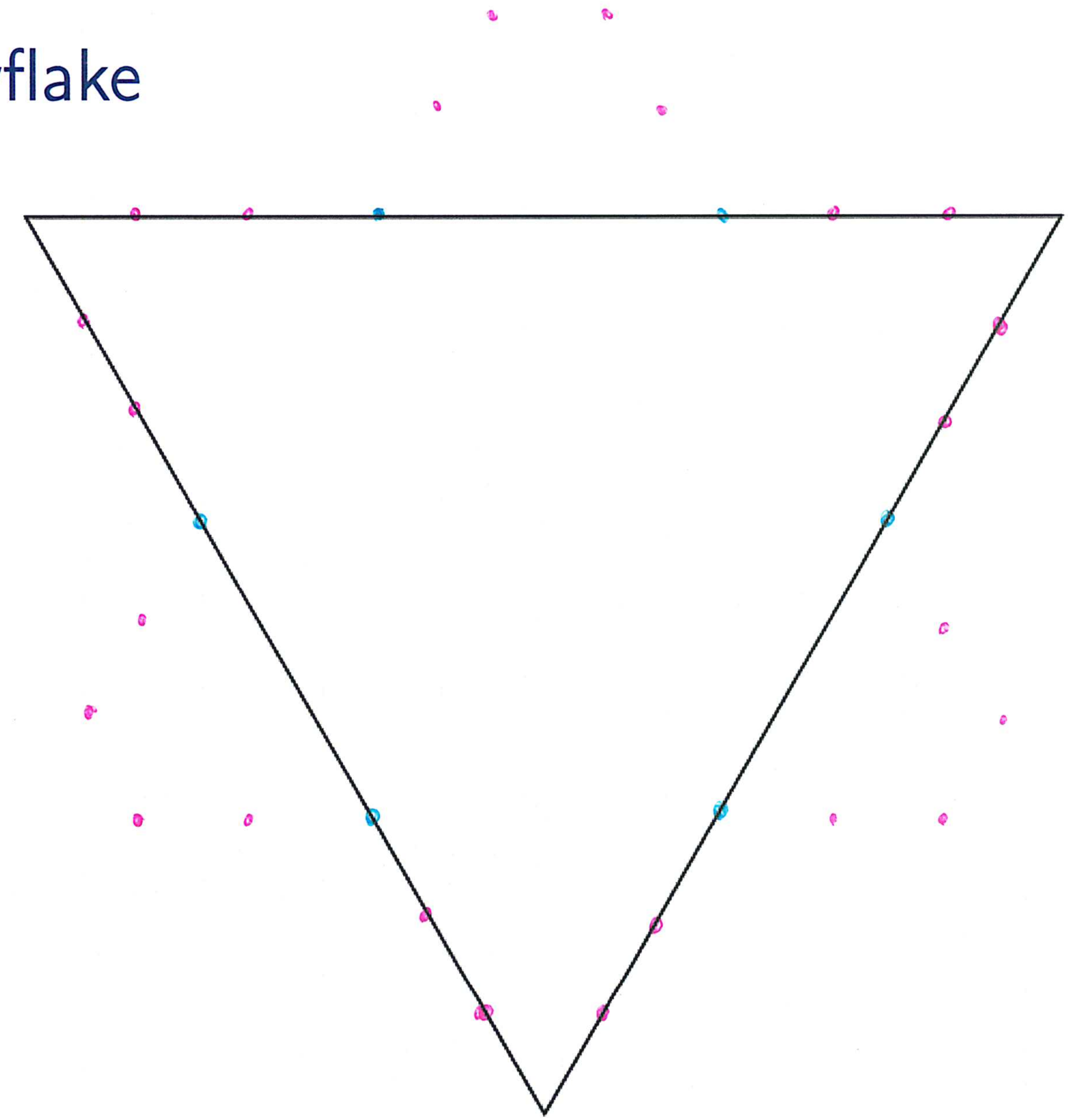


Example



Fractal Tree

Koch Snowflake



Definitions (page 124 of text)

- A set of points in the plane is **bounded** if

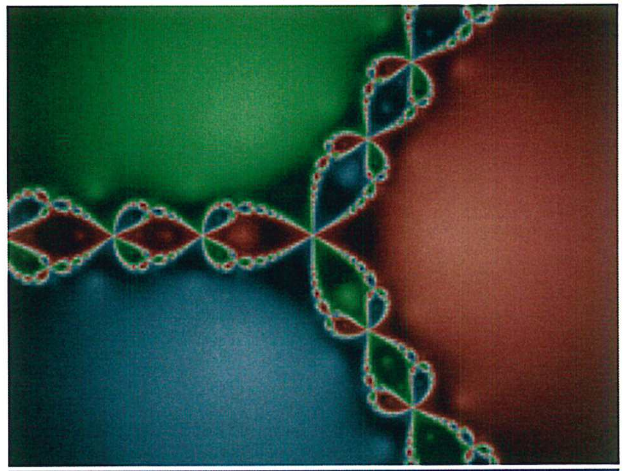
- A set of points in the plane is **unbounded** if the set is not bounded.

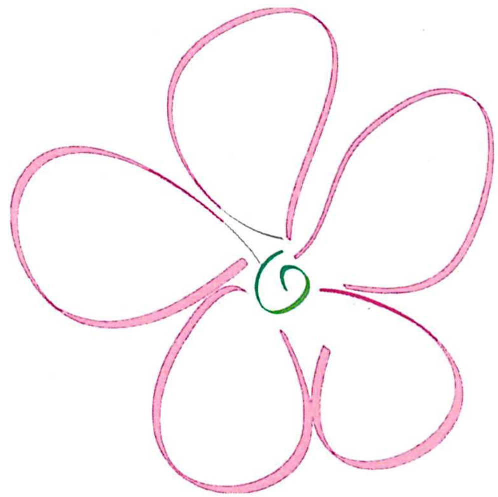
Juliet Sets – Definitions (page 125 of text)

Let f be a transformation of the points in the plane.

- The **prisoner set** (or **filled Julia set**) is the set of points A where $\{A, f(A), f(f(A)), \ldots\}$ is
- The **escape set** is the set of points A where $\{A, f(A), f(f(A)), \ldots\}$ is
- The **Juliet set** is the boundary between the prisoner set and the escape set.

Juliet Sets In Colour!





QUESTIONS???