MATH 1020/FA 1020 A01: Mathematics In Art
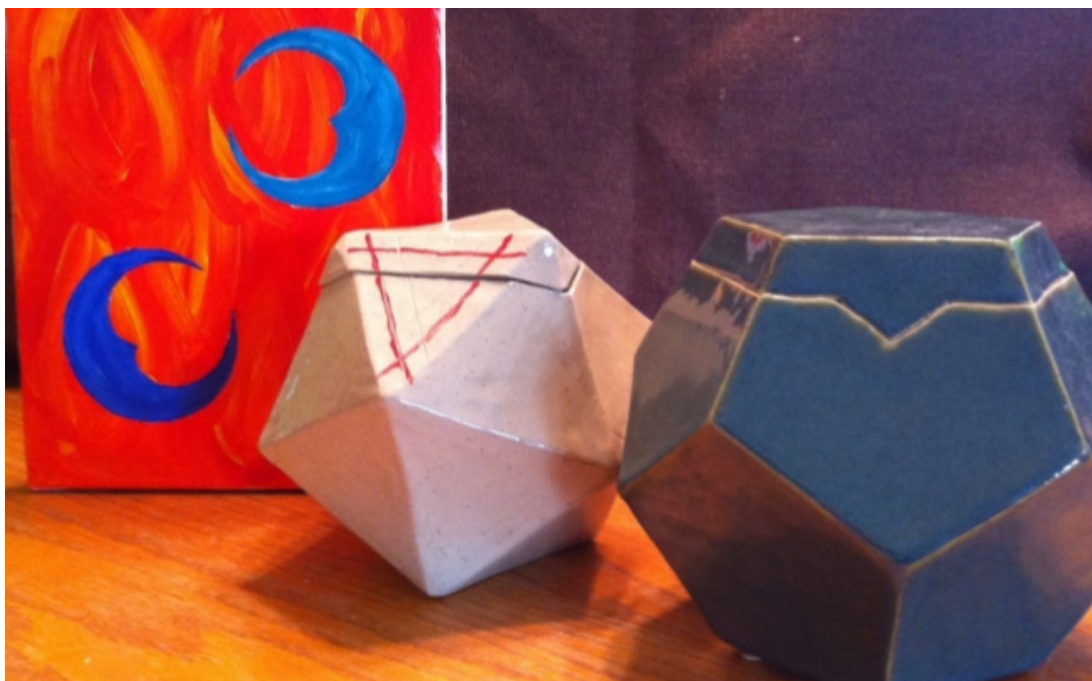## Course Information Sheet and Syllabus
Summer 2019

BASIC COURSE DETAILS

Instructors: Dr. Susan M. Cooper (Math Part) & Shaun M. De Rooy (Art Part)

Class Times and Location: Tuesdays & Thursdays 9:30 a.m. – 12:00 p.m., 136 Art Lab

Credit Hours: 3

Pre-Requisites and Notes: No pre-requisites. The course will be team-taught by faculty from the School of Art and the Department of Mathematics. This is a terminal course and may not be used as a pre-requisite for other Mathematics courses. This course cannot be used as part of an Honours, Major, General or Minor program in the mathematical sciences. Not available to any student already holding a grade of "C" or better in any Mathematics course with the exception of MATH 1010, the former MATH 1190, or MATH 1191. Not to be taken concurrently with any other Mathematics course with the exception of MATH 1010 or MATH 1191. FA 1020 is not to be held for credit with MATH 1020.

MATHEMATICS INSTRUCTOR CONTACT INFORMATION

Instructor: Dr. Susan Cooper   **Email:** susan.cooper@umanitoba.ca

Office: Machray Hall, Room 464   **Office Phone:** 204-474-9701

Office Hours: Tuesdays & Thursdays 1:00 p.m. – 2:00 p.m.; or by appointment

Correspondence and Appointments: The most reliable way to contact me is via email. I will reply to an email within 24 hours of receiving it Mondays – Thursdays; an email received on a Friday will receive a reply the following Monday. All appointments are to be made via email.

MATHEMATICS COURSE DESCRIPTION AND GOALS

"There is an art to science, and science in art; the two are not enemies, but different aspects of the whole." — Isaac Asimov

University of Manitoba Course Calendar Description: Specific theory, structuring systems, and mathematical methods and principles used in works of art from various historical periods and contexts will be explored in relation to Euclidean and non-Euclidean geometries. Topics include: linear perspective; shapes, patterns, balance and symmetry; ratio, proportion and harmony; and order, dynamics, and chaos. The course will be one half art and one half mathematics, team-taught by faculty from the School of Art and the Department of Mathematics. This course is also given in the School of Art as FA 1020.

General Course Description and Goals: At the outset, mathematics and art may seem to be very disconnected. However, the two disciplines are indeed very much interconnected. Underlying many visually appealing objects in both art and nature is mathematics. Indeed, the picture on page 1 demonstrates actual mathematics in art made by Dr. Michelle Davidson, Associate Head (Undergraduate Studies) of the Department of Mathematics here at the University of Manitoba. Mathematics in itself is a beautiful language and science that has played a critical role in the creation of our modern world. In this course you will gain familiarity with some mathematical tools and ideas that can be used in creating and understanding art. We will look at historical milestones in which mathematics has played an important role in art. In our explorations we will focus on looking for patterns, finding hidden underlying structures, and experiencing ideas in as many ways as possible. The main themes of study include: constructions in Euclidean geometry, golden mean, golden rectangles, Fibonacci spirals, symmetries and other organizing principles, frieze patterns, wallpaper groups, tilings & tessmelations, fractals, string art and conics, perspective drawing, Platonic solids and regular polyhedra, Escher-style hyperbolic art, and isotopy and homotopy of topological objects.

MATHEMATICS COURSE MATERIALS

Textbook: *Math and Art: An Introduction to Visual Mathematics*, by Sasho Kalajdzievski and R. Padmanabhan

[**Note:** All of the royalties for the books sold in the University of Manitoba bookstore go back to students through scholarship funds.]

Course Web-Page: We will use the instructor's web-page which can be found at

`http://server.math.umanitoba.ca/~coopers5/courses_umanitoba/math1020_summer19.html`

and UM Learn.

WebMathematica Page: This page contains applets for real-time manipulation and drawing of various objects (fractals, tilings, etc.) and can be found at

`http://webware.cc.umanitoba.ca:8080/webMathematica/Files/MathInArt.html`.

Additional Geometry Tools: You will want a compass and a straight edge (ruler) for this course. You may also use a calculator from time-to-time.

ASSESSMENTS AND COURSE GRADES

Mathematics Assignments: Mathematics is not a spectator sport. The best way to learn mathematics is by doing mathematics! You are highly encouraged to work through sample exam questions which will be shared throughout the term. Moreover, a collection of suggested problems and readings from the mathematics textbook is listed in the table below. Although your solutions will not be formally collected for credit, you are encouraged and welcome to discuss your solutions with me for feedback either in office hours or before/after class meetings. The sample exam questions and suggested problems below are intended to gauge your understanding of the material while presenting opportunities for you to practice reading and communicating mathematics. Note that you should attempt the sample exam questions and suggested problems regularly as the topics are presented – this will help you keep on track and make sure you have solid foundations before moving on to the next topic.

Textbook Section Readings	Pages	Suggested Problems
1.1 Euclidean Geometry	1–6	
1.2 Euclidean Constructions	6–14	1–8
1.3 Golden Ratio	14–24	1–11
1.4 Fibonacci Numbers	24–31	1–6
2.1 Plane Symmetries	33–42	1–9
2.3 Group of Symmetries	55–60	1–7
2.4 Frieze Patterns (part)	61–72	1–3
2.5 Wallpaper Designs; Tilings (part)	72–81	
2.6 Tilings and Art (part)	81–89	
3.1 Similarities	91–100	1–7
3.3 Fractals (part)	100–123	1–4
3.4 Julia Sets (part)	123–131	1–3
4.1 Non-Euclidean Geometries	143–146	
4.2 Inversion	146	
4.3 Hyperbolic Geometry	153–158	
4.4 Hyperbolic Constructions	158–163	1–7
4.5 Tilings in Hyperbolic Plane (part)	163–167	
5.1 Perspective	169–181	1–9
5.3 Polyhedra (part)	197–206	1–4
5.4 Conic Sections (part)	206–216	1–6
6.1 Homotopy	223–230	1–6
6.2 Two-Manifolds and Euler (part)	230–237	1–6
6.3 Other Manifolds (overview only)	237–247	

Art Projects: In addition to the sample mathematics exam questions and suggested problems, you will be assigned Art projects. The format, deadlines, and late submission policies for the Art projects will be determined by the Art Instructor, Shaun M. De Rooy.

Examinations: There will be one mid-term examination and one 2-hour cumulative final examination (set by Dr. Cooper). The schedule is:

Examination	Date	Time and Location
Mid-Term Exam	Tuesday, May 28	9:30 – 10:45 a.m., in class
Final Exam	Saturday, June 22	1:30 – 3:30 p.m., Robert Schultz Lecture Theatre

Resources permitted and grading rules for the examinations will be announced and discussed prior. Solutions to problems will be assessed based on correctness, clarity, and style. Feedback on your solutions will be provided on your mid-term examination and is meant to improve your mathematical abilities and communication. Grades and feedback on your mid-term examination solutions will be provided prior to the Voluntary Withdrawal Deadline.

Deferred Examinations: Deferred mid-term examinations will only be granted for unavoidable, documented circumstances as described below:

Circumstance	Required documentation
Illness or other medical situation	Official note from clinic, hospital, doctor, nurse, or other health care provider
Military service	Official military activation orders
Funeral or other family emergency	Official documentation from newspaper, funeral, or medical official
Sports or other official U of M activity	Official documentation from U of M athletics or activity's faculty adviser

Students who wish to request a deferred mid-term examination must contact Dr. Cooper within 24 hours of the scheduled examination (initial email contact is sufficient). Please note that recreational activities do not qualify for deferred examinations. If you have a pre-existing conflict with an examination, you are expected to make alternative arrangements *beforehand*.

As per university policy, requests for deferred final examinations are made to the student's faculty, school, or academic advising office and must be filed within 48 hours of the date of the missed examination. Full information on this policy can be found at:

<div align="center">http://umanitoba.ca/student/records/finals/682.html</div>

Class Attendance and Participation: This course covers a large amount of material and may seem very fast-paced. Your understanding of the course material will be greatly supported by regular attendance and engagement in class meetings. Although you are expected to attend every class meeting, attendance will not be taken or be used in the calculation of course grades. However, you are responsible for any missed material when absent. If time permits, we will discuss some of the suggested mathematics problems. In such discussions, students will be asked to share their ideas. Please take your turn in these activities – it will greatly improve your understanding of the material. In particular, if you are absent from class then you will miss the opportunity to learn from your classmates.

Evaluation Scheme: Final course grades will be determined by the following scheme:

Task	Percentage of Grade
Art Projects	40%
Mid-Term Exam	25%
Final Exam	35%

Letter Grades: The letter grade cut-offs listed below show the minimum cut-off ranges for the course. These cut-offs may change (decrease) at the instructors' discretion.

Letter Grade	Percentage Out Of 100	Grade Point Range	Final Grade Point
A+	95–100	4.25–4.5	4.5
A	86–94	3.75–4.24	4.0
B+	80–85	3.25–3.74	3.5
B	72–79	2.75–3.24	3.0
C+	65–71	2.25–2.74	2.5
C	60–64	2.0–2.24	2.0
D	50–59	Less than 2.0	1.0
F	Less than 50		0

MATHEMATICS COURSE SCHEDULE AND IMPORTANT DATES

Below is a *tentative* overall schedule of topics and dates for our course. This schedule is subject to change at the discretion of the instructor and/or based on learning needs of the students but such changes are subject to Section 2.8 of ROASS (Responsibilities Of Academic Staff With Regard To Students).

DATE	TOPIC
May 7	Course Overview; Triangle Notes; Euclidean Constructions
May 9	Euclidean Constructions; Golden Ratio
May 14	Golden Ratio; Fibonacci Sequence
May 16	Fibonacci Sequence; Symmetries
May 21	Symmetries; Groups of Symmetries; Friezes
May 23	Friezes; Tilings; Wallpapers; Mid-Term Review (time-permitting)
Mid-Term Examination: Tuesday, May 28 (In Class)	
May 30	Similarities; Fractals
June 4	Fractals; Perspective
June 6	Conic Constructions; Platonic Solids
June 11	Platonic Solids; Hyperbolic Geometry
Voluntary Withdrawal Deadline: Wednesday, June 12	
June 13	Hyperbolic Geometry
June 18	Topology
June 20	Topology; Final Exam Review (time-permitting)
Final Examination: Sat., June 22, 1:30 – 3:30 p.m., Robert Schultz Lecture Theatre	

UNIVERSITY AND DEPARTMENT OF MATHEMATICS SUPPORT OFFICES AND POLICIES

A list (entitled Schedule "A") of supports available to students, including mathematical support, can be found on the course web-page(s).

MATHEMATICS COURSE WORKSHOPS AND EXTRA SUPPORT

The Department of Mathematics has arranged special support for students needing help with the mathematics portion of the course. In particular, a TA who has special expertise with the course material will work in the Mathematics Help Centre on Mondays 2:00 – 5:00 p.m. Details about the Mathematics Help Centre can be found on Schedule "A" posted on the course web-page(s). In addition, a TA will offer a number of *MATH 1020/FA 1020 Workshops.* During these optional workshops students can receive extra assistance and practice with some of the mathematics material of the course. The dates, times, and locations of the workshops are listed below. Note that there are nearly two workshops hosted every week and that within a given week the topics will remain the same. Topics of the workshops will be announced as the course progresses.

DATE	TIME	LOCATION
Monday, May 13	10:00 – 11:15 a.m.	240 University College
Wednesday, May 15	5:30 – 6:45 p.m.	240 University College
Wednesday, May 22	5:30 – 6:45 p.m.	240 University College
Monday, May 27	10:00 – 11:15 a.m.	240 University College
Monday, June 3	10:00 – 11:15 a.m.	240 University College
Wednesday, June 5	5:30 – 6:45 p.m.	240 University College
Monday, June 10	10:00 – 11:15 a.m.	240 University College
Wednesday, June 12	5:30 – 6:45 p.m.	240 University College
Monday, June 17	10:00 – 11:15 a.m.	240 University College
Wednesday, June 19	5:30 – 6:45 p.m.	240 University College

MATHEMATICS COURSE EXPECTATIONS

Recording Class Lectures: Susan Cooper and the University of Manitoba hold copyright over the course materials, presentations, and lectures which form part of this course. No audio or video recording of lectures or presentations is allowed in any format (including photographs), openly or surreptitiously, in whole or in part without permission. Course materials (both paper and digital) are for the participant's private study and research.

Using Copyrighted Material: Please respect copyright. We will use copyrighted material in this course. I have ensured that the content I use is appropriately acknowledged and is copied in accordance with copyright laws and University guidelines. Copyrighted works, including those created by me, are made available for private study and research and must not be distributed in any format without permission. Do not upload copyrighted works to a learning management system (such as UM Learn), or any website, unless an exception to the *Copyright Act* applies or written permission has been confirmed. For more information, see the University's Copyright Office web-site at `http://umanitoba.ca/copyright/` or contact um_copyright@umanitoba.ca.

Course Technology: It is the University of Manitoba policy that all technology resources are to be used in a responsible, efficient, ethical and legal manner. During class meetings and for course assignments/exams, it is expected that you only use technology for educational purposes and that the only technology used is approved by myself and/or the University of Manitoba Student Accessibility Services. You should not participate in personal direct electronic message/posting activities (such as e-mail, texting, video, social networking, etc.) during scheduled class time – this is not only in your best interests for understanding the course material but is respectful behaviour for your classmates. If you absolutely need to take an expected call, then please use the vibrate mode on your cell phone and leave the classroom before using the phone.

Class Communication: You are required to obtain and use your University of Manitoba email account for all communication between yourself and the university. All communication must comply with the Electronic Communication with Student Policy.

Student Accessibility Services: The University of Manitoba is committed to providing an accessible academic community. *Students Accessibility Services (SAS)* offers academic accommodation supports and services such as note-taking, interpreting, assistive technology and exam accommodations. Students who have, or think they may have, a disability (e.g. mental illness, learning, medical, hearing, injury-related, visual) are invited to contact SAS to arrange a confidential consultation. Students are welcome to meet with the instructors to discuss the accommodations recommended by SAS.

> Student Accessibility Services
> http://umanitoba.ca/student/accessibility/index.html
> 520 University Centre
> Phone: (204) 474–7423
> Email: Student_accessibility@umantioba.ca

Academic Integrity: You are expected to be academically honest. This means, for example, providing a list of the people (if any) with whom you worked and providing a list of sources other than the textbook (if any) that you used to complete an assignment. Although you are encouraged to work together, you should never submit anything that you do not understand or is not written in your own words. The following excerpt about Academic Honesty is taken from the Department of Mathematics web-page:

The Department of Mathematics, the Faculty of Science and the University of Manitoba regard acts of academic dishonesty in quizzes, tests, examinations or assignments as serious offences and may assess a variety of penalties depending on the nature of the offence. Acts of academic dishonesty include bringing unauthorized materials into a test or exam, copying from another student, plagiarism and examination personation. Students are advised to read the sections entitled "Academic Integrity" and "Final Examinations" (in particular, the section "Examination Personations") in the "General Academic Regulations" section of the current Undergraduate Calendar.

Penalties for violation include being assigned a grade of zero on a test or assignment, being assigned a grade of "F" in a course, compulsory withdrawal from a course or program, suspension from a course/program/faculty or even expulsion from the University. For specific details about the nature of penalties that may be assessed upon conviction of an act of academic dishonesty, students are referred to University Policy 1202 (*Student Discipline Bylaw*) and to the Department of Mathematics policy concerning minimum penalties for acts of academic dishonesty. Information regarding cheating and plagiarism is also available from the Faculty of Science web-page.

Students are encouraged to visit the University of Manitoba Academic Integrity site for further information.

Classroom Atmosphere: A part of learning is making mistakes. We want to establish a classroom atmosphere where the inevitable false starts and mistakes become an opportunity to improve – not an opportunity for embarrassment. Please be constructive and polite in questioning your colleagues.

Other Expectations and Tips for Success: I ask that you have a well-defined sense of professionalism, that you always put forth your best effort, and that you develop a sense of responsibility to your educational community. I ask that you exhibit a persistent desire to learn. In return I will provide you with significant support. Also:

- Be an active participant - mathematics is learned by doing; this includes participating fully in classroom activities completing the suggested problems, critically thinking about the mathematics and art during and outside of class. *In order for this class to be successful, it is imperative that you commit to coming to class regularly, that you commit to coming to class prepared, and that you commit to participating in class!*

- Be positive, open, and responsive to feedback.
- Be/become a "risk taker".
- Be committed.
- Be patient with yourself - it takes time to master newly learned things. Ask for assistance when it is needed. Constantly try to improve yourself as a mathematician.
- Starting with the first class, study in-depth and regularly. This means, for example, that you should do the suggested problems *before* the next class period.
- It is tempting to just copy available solutions. However, struggling through the exercises on your own is an important phase of the learning process.
- Get help as soon as you need it: ask questions in class and office hours; form a study group; attend the MATH 1020/FA 1020 Workshops; consider getting a tutor, etc.
- For exam preparation, practice exercises that have not been assigned.
- Save your solutions to suggested problems to study from. You may find it helpful to organize your work in a 3-ring binder or notebook for your suggested problems. You should be able to look at the solutions a month later and understand what is on the paper.
- Mathematics is a language in itself that is common to many sciences across the world. It is crucial that we all use consistent and correct notation. For example, when using the equal sign you should make sure that the quantities on either side of the equal sign are indeed equal.
- Take pride in your work and take your work seriously. This means you should: use complete sentences with proper grammar and correct spelling, write legibly, provide justification for your claims, show all of your work, clearly state all the hypotheses being used, etc.
- Like in all areas of life, constructive feedback can be difficult to digest and accept. Please know that the feedback provided in this course is meant to *improve* your mathematical solutions and communication. Please take the feedback seriously and apply it to your future work.
- Everyone wants you to succeed. Please speak with me regarding any concerns you may have.
- Relax and have fun with the course!