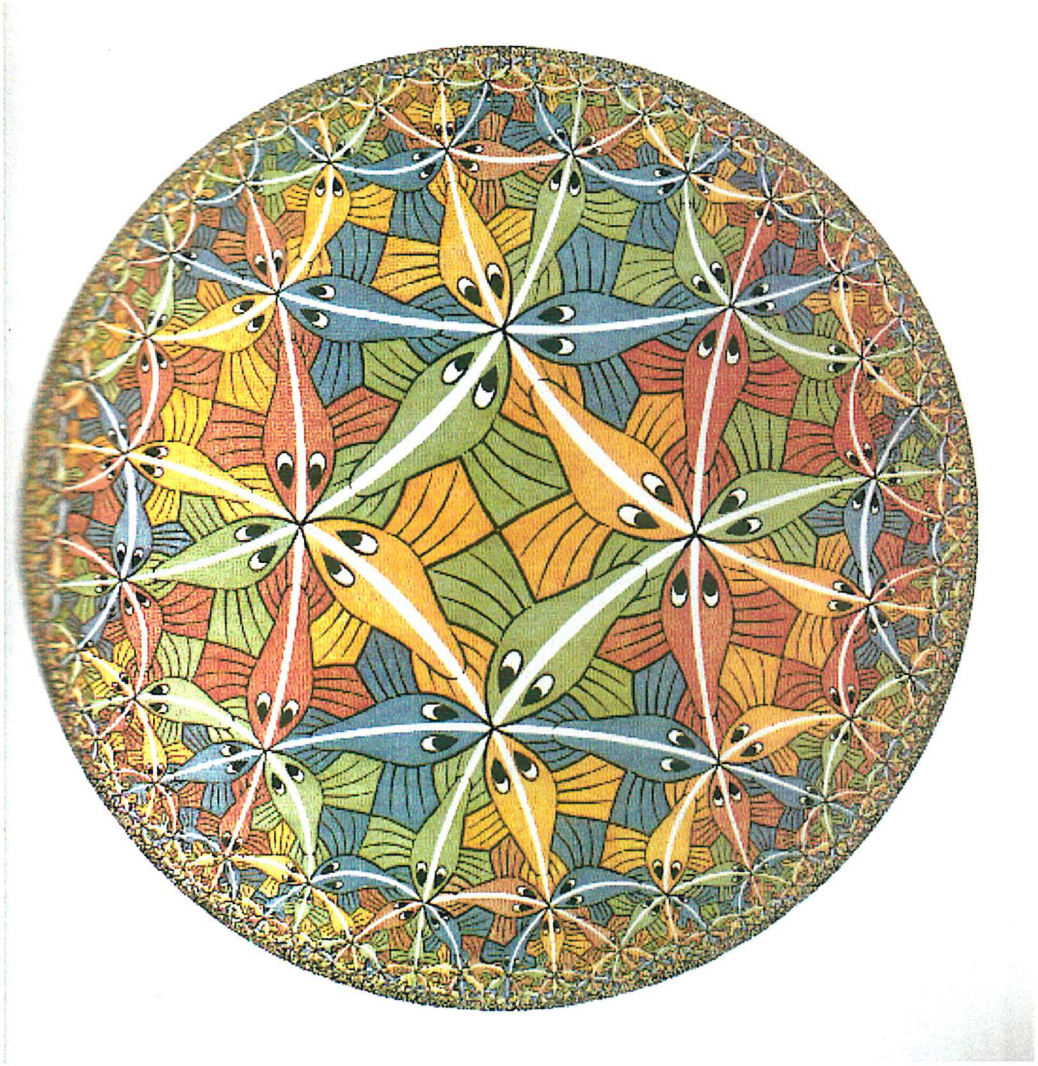
Hyperbolic Geometry



Recall: Euclid's Postulates For Euclidean Geometry

- There exists a line through distinct points P and Q.
- Line segments can be extended.
- Circles exist.
- All right angles are congruent.

Euclid's 5th Postulate

For every line ℓ and a point P that does not lie on ℓ, there exists an unique line m through P and parallel to ℓ.

Note: There are two ways that we can change this postulate:

- replace "an unique" with
- replace "an unique" with

Hyperbolic 5th Postulate

Given a line ℓ and a point P not on ℓ, there are **many** lines through P and parallel to ℓ.

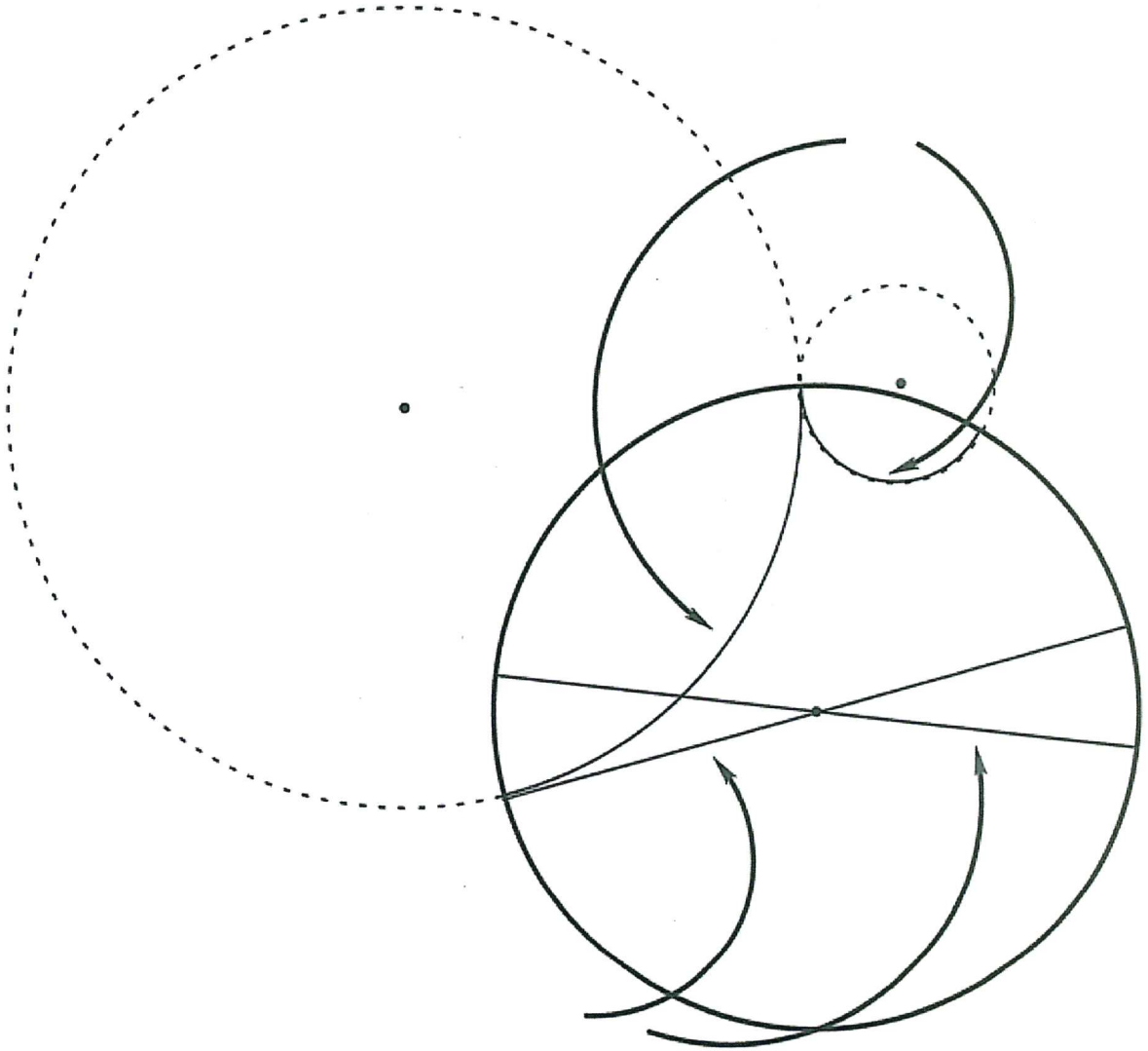
****Note: In this context, parallel MEANS**

Poincaré Model Of A Hyperbolic Geometry

Given a circle H with center O:

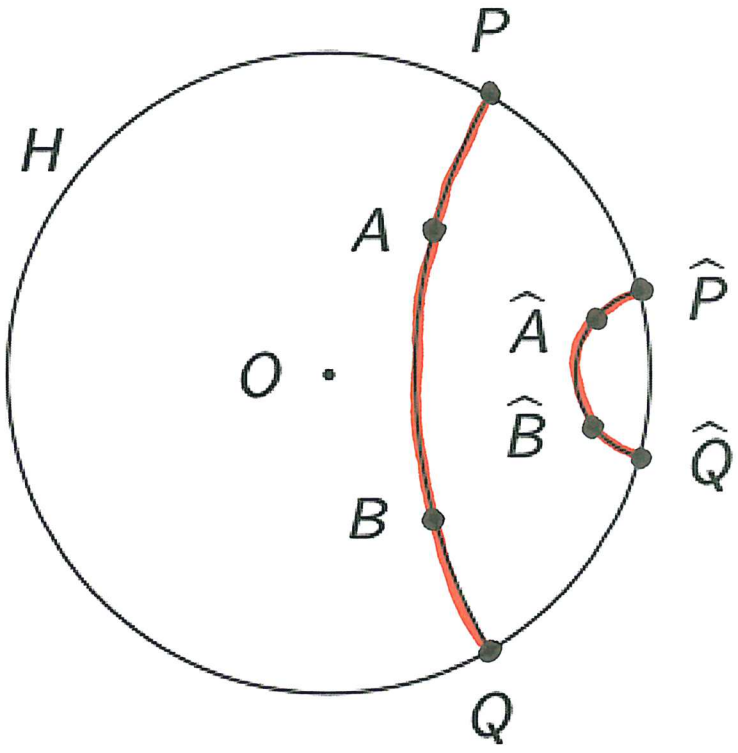
- The points of the geometry are all the points that are
- Lines of the geometry are of two types:
  - diameters;
  - parts of circles that are

Poincaré Model



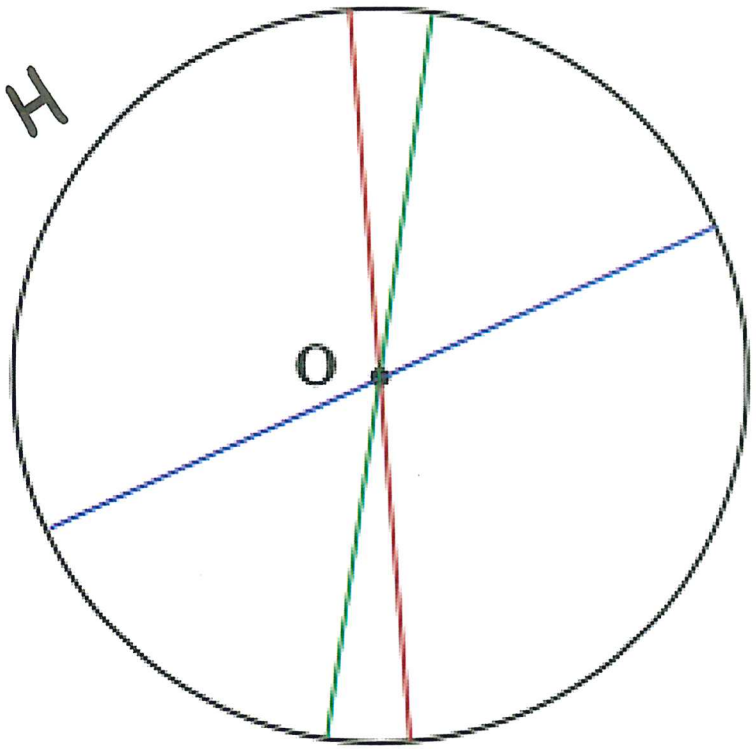
Hyperbolic Distance

The hyperbolic distance between two hyperbolic points A and B is determined by a ratio of distances between A and B and the points P and Q on the hyperbolic horizon on the unique line joining A and B.

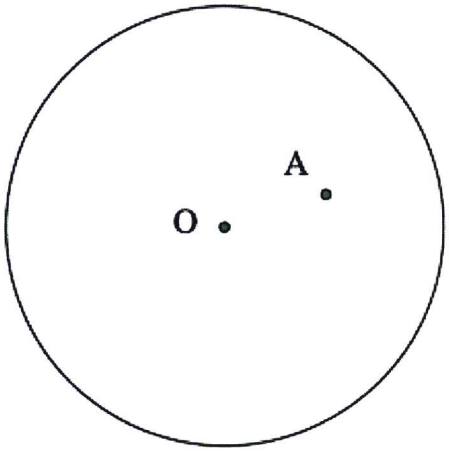


Construction 1: Hyperbolic Lines Through Center O of H

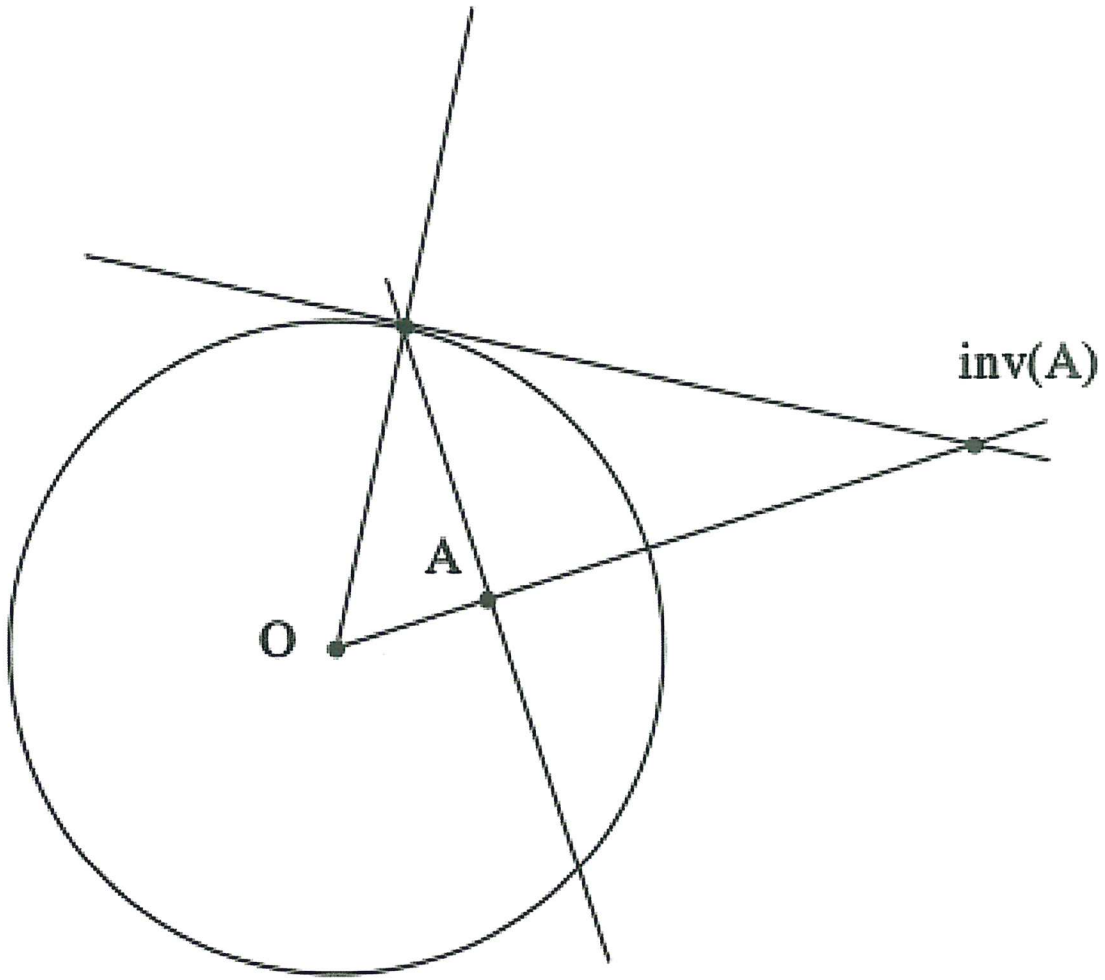
All hyperbolic lines that pass through O are



Preliminary For Hyperbolic Lines Not Through Center: Circle Inversion



Circle Inversion



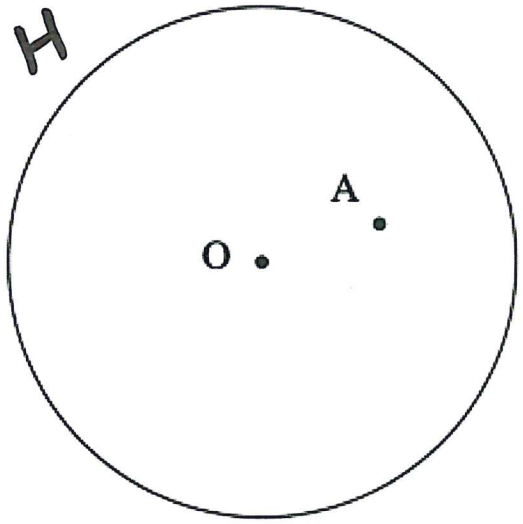
Construction 1: Hyperbolic Lines Through $A \neq O$

Definition: The perpendicular bisector of the line $Ainv(A)$ is called the

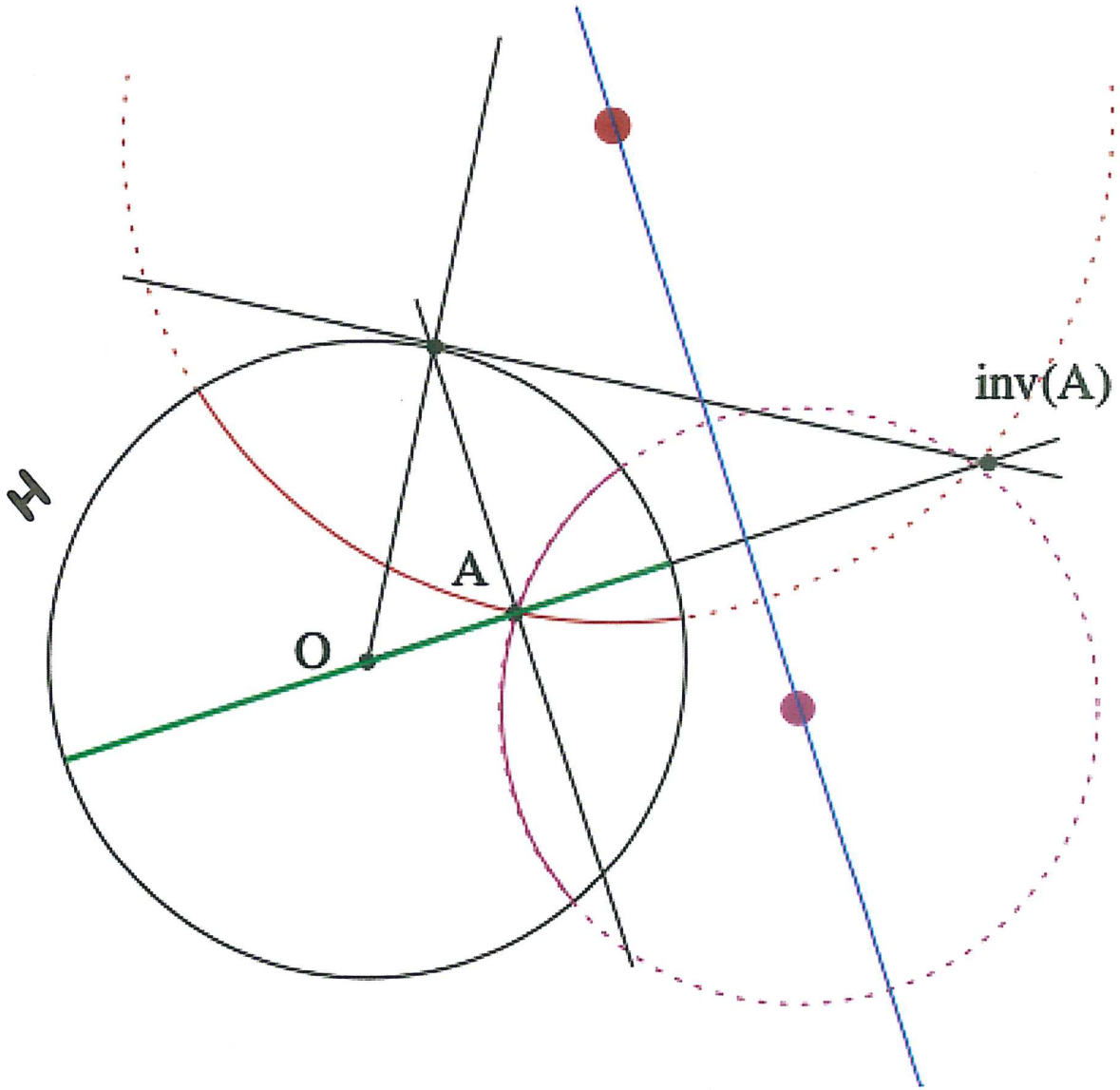
All of the lines through A are:

- the diameter that passes through A;
- the part of the interior to H of a circle that has center on ℓ_A and passes through

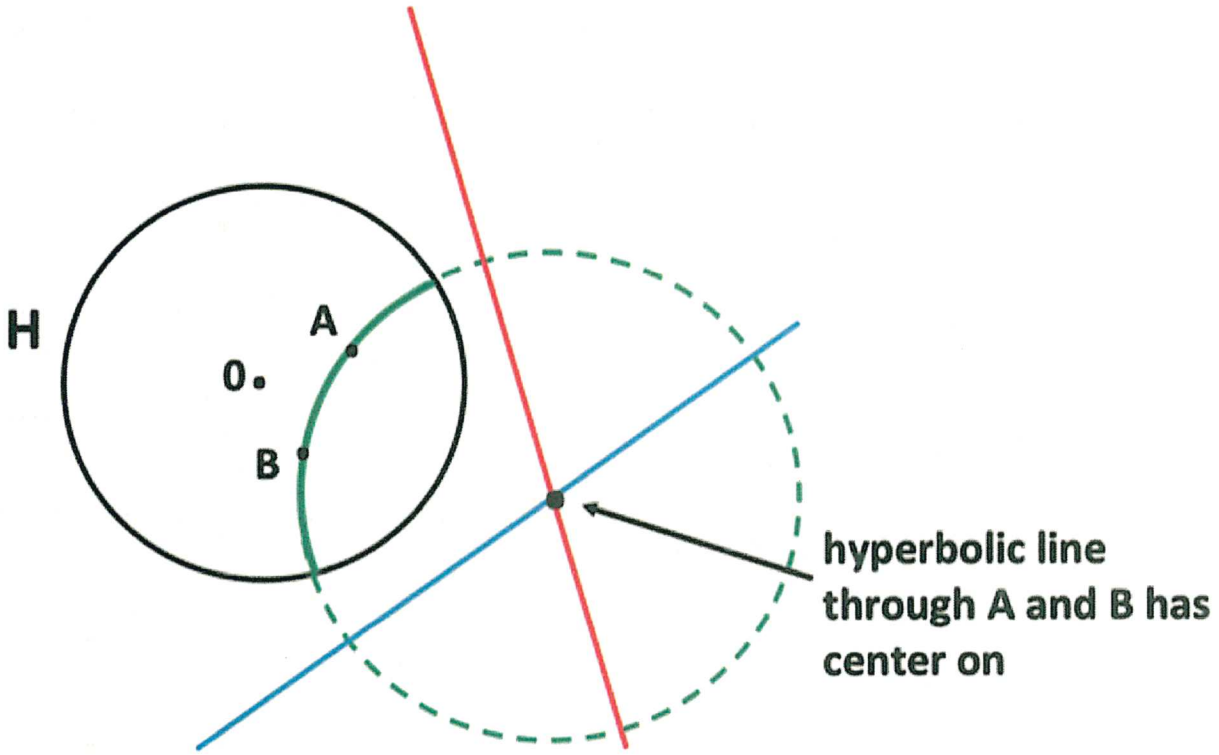
Construction 1: Hyperbolic Lines Through $A \neq O$



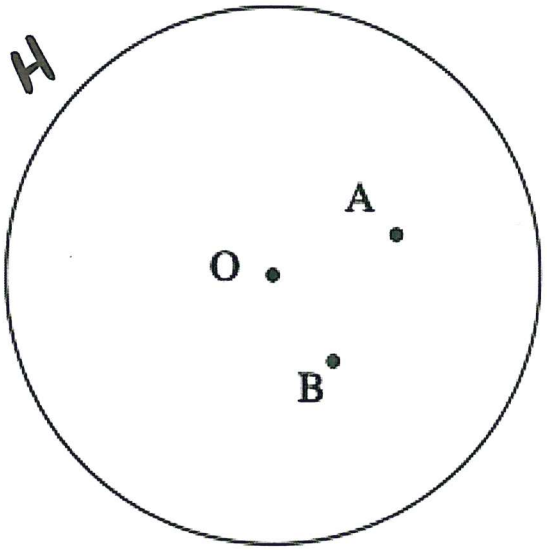
Construction 1: Hyperbolic Lines Through $A \neq O$



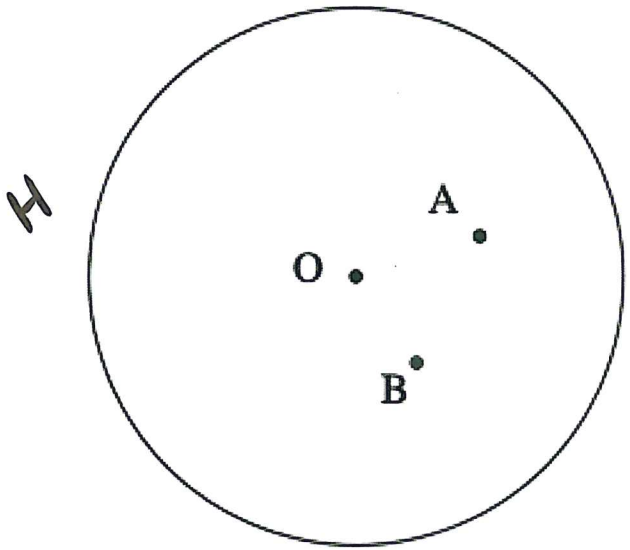
Construction 2: Hyperbolic Line Passing Through *A* & *B* (Main Idea!)



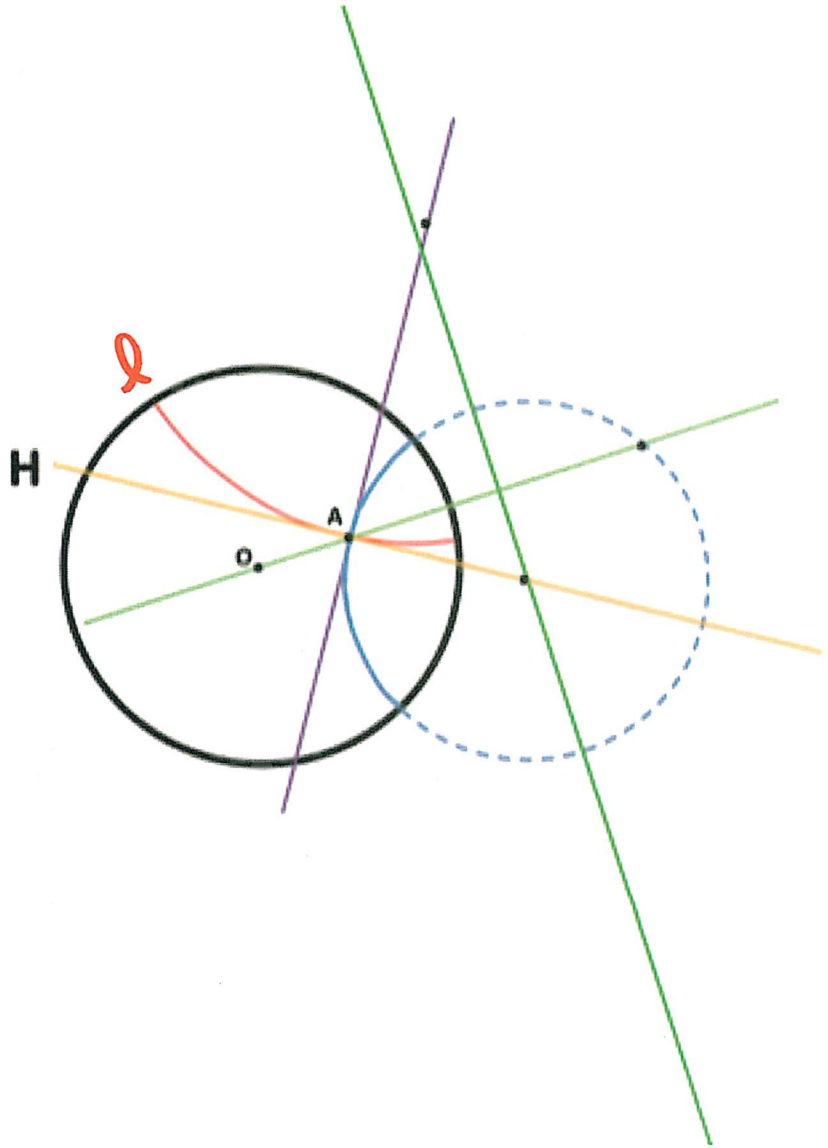
Construction 2: Hyperbolic Line Passing Through *A* & *B*



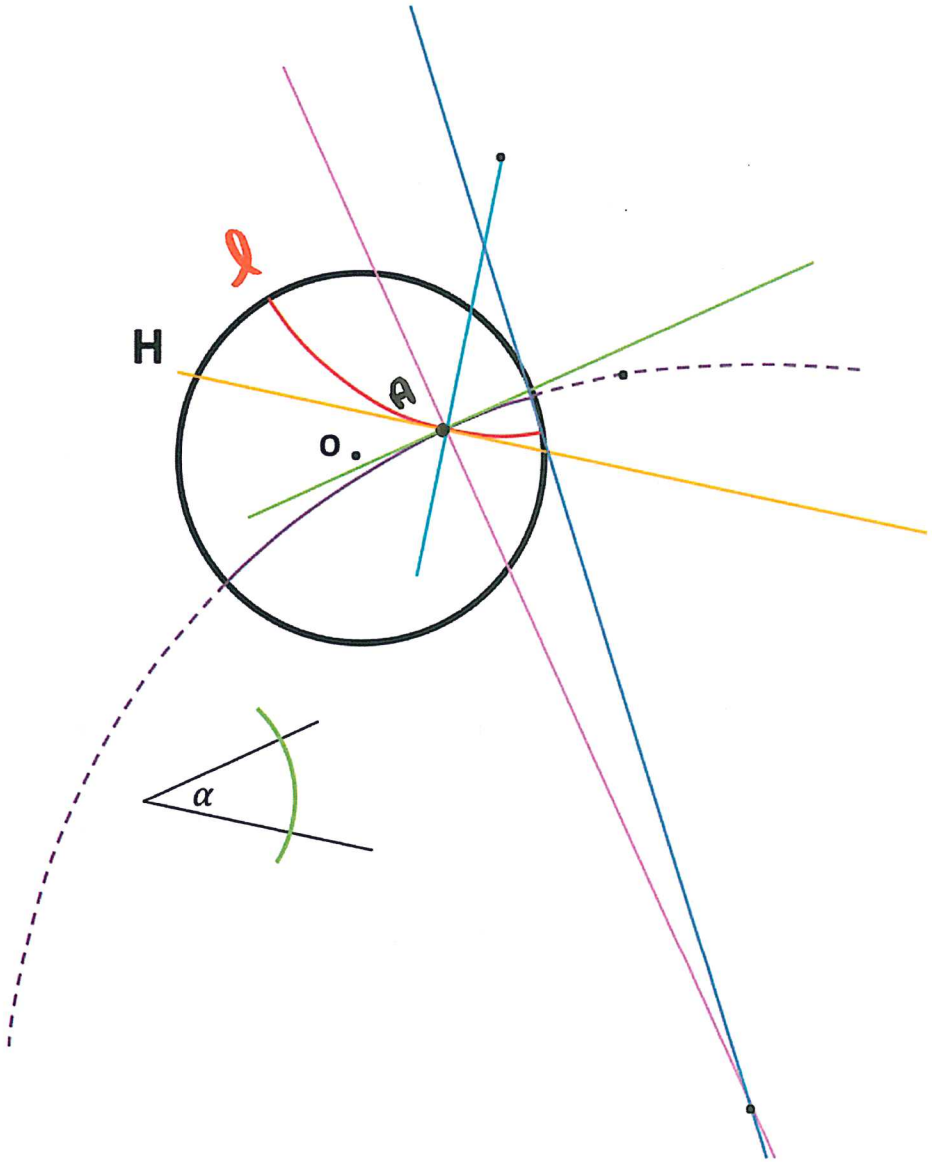
Alternate Construction 2: Hyperbolic Line Through *A* & *B*



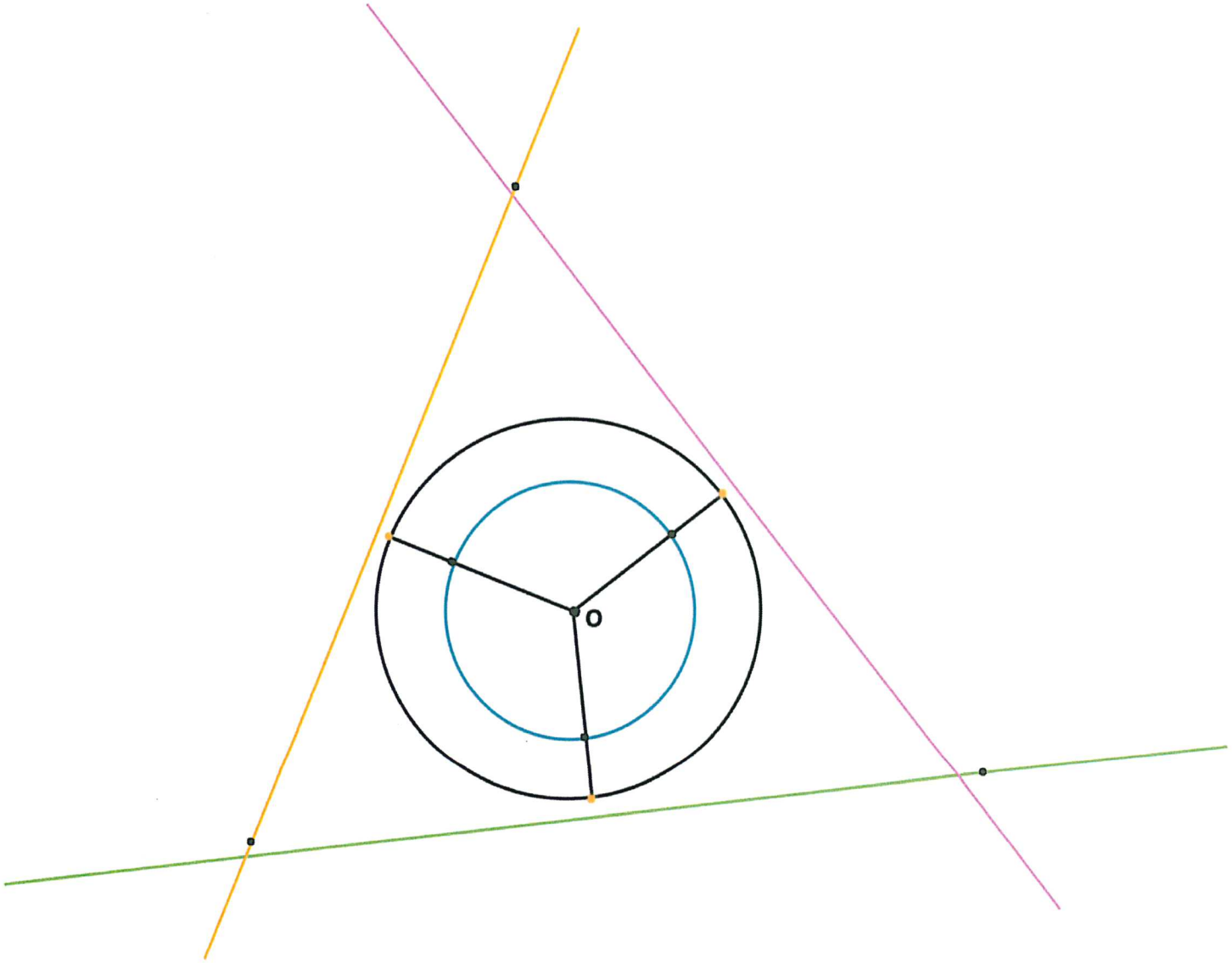
Construction 3: Intersecting Hyperbolic Lines At 90° Angle



Constr. 3: Intersecting Hyperbolic Lines At Given Angle

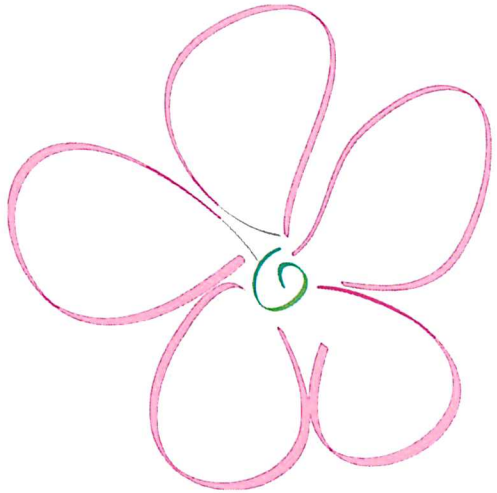


Construction 4: An Equilateral Triangle



Sum Of The Angles Of A Triangle

In a hyperbolic plane, the sum of the angles of a triangle is



QUESTIONS???