Leonardo Pisano (Fibonacci)

The Classic Problem

From *Liber Abaci* by Pisano Fibonacci (around AD 1200; also introduced the Hindu-Arabic numeral system to Western Europe):

A certain man puts a pair of rabbits in a place surrounded on all sides by a wall. How many pairs of rabbits can be produced from that pair in a year if it is supposed that every month each pair begets a new pair, which from the second month on become productive?

- Implicitly assuming that no rabbit dies!

The Rabbit Problem: The First Few Months

Month	1	2	3	4	5	6	7	8
Total Pairs	1	1	2					

Some Notation and Observations

- Let f_n be the number of pairs of rabbits after n months.

We have:

The Recursive Definition

The **Fibonacci Numbers** are the numbers in the sequence defined by

$$f_1 = 1$$
$$f_2 = 1$$
$$f_n = f_{n-1} + f_{n-2}$$

Example With Recursive Definition

Given that $f_{19} = 4181$ and $f_{16} = 987$, what are f_{17} and f_{18}?

An Explicit Formula

Binet's Formula for the Fibonacci Numbers:

$$f_n = \frac{(1+\sqrt{5})^n - (1-\sqrt{5})^n}{2^n\sqrt{5}}$$

An Approximation

- Recall: $\varphi = \dfrac{1+\sqrt{5}}{2} \approx 1.618$ is the Golden Ratio.

Look at what happens as $n \to \infty$:

$$f_n \approx \frac{\varphi^n}{\sqrt{5}}$$

A Cute Conversion

- $\varphi \approx$ the number of kilometers in a mile! (Exact $= 1.609344$.)

E.g.: Convert 30 kilometers to its equivalent in miles:

$$30 = 21 + 8 + 1 = f_8 + f_6 + f_2 \approx \frac{\varphi^8}{\sqrt{5}} + \frac{\varphi^6}{\sqrt{5}} + \frac{\varphi^2}{\sqrt{5}}.$$

Then the number of miles in 30 kilometers would be approximately

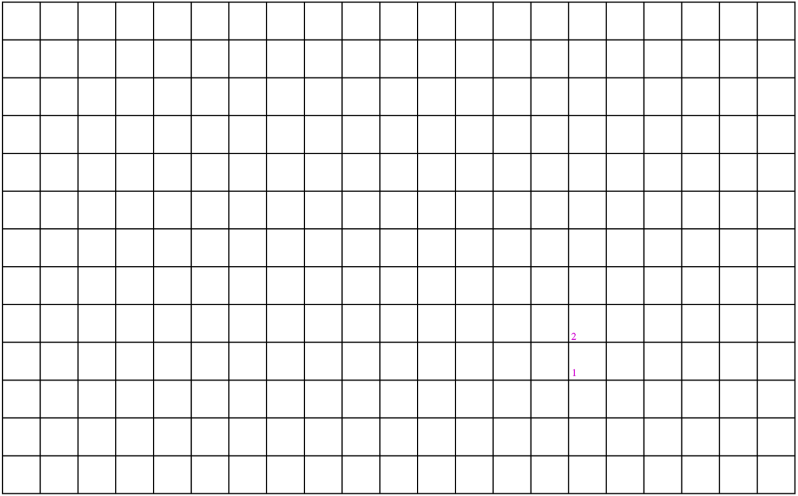
$$\frac{30}{\varphi} \approx \frac{\varphi^7}{\sqrt{5}} + \frac{\varphi^5}{\sqrt{5}} + \frac{\varphi}{\sqrt{5}} \approx f_7 + f_5 + f_1 = 13 + 5 + 1 = 19.$$

The actual number of miles in 30 kilometers is 18.64.

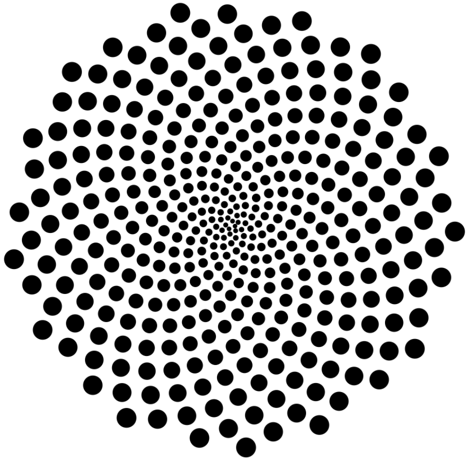
Ratios of Consecutive Fibonacci Numbers

Ratio of Fibonacci Numbers	Ratio	Decimal Equivalent
f_2/f_1	1/1	
f_3/f_2	2/1	
f_4/f_3	3/2	
f_5/f_4	5/3	
f_6/f_5	8/5	
f_7/f_6	13/8	
f_8/f_7	21/13	
f_9/f_8	34/21	
f_{10}/f_9	55/34	
f_{11}/f_{10}	89/55	

Fibonacci Spiral

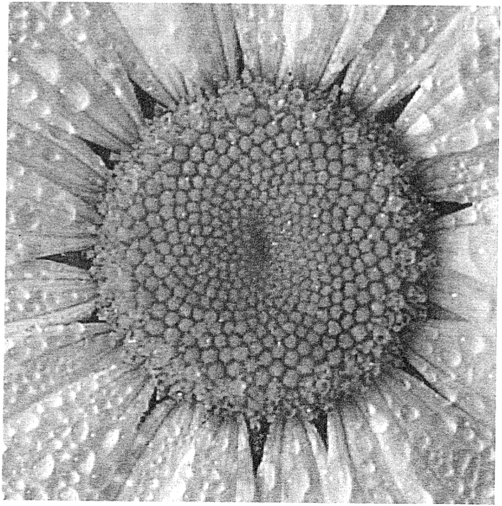


Fibonacci Flowers



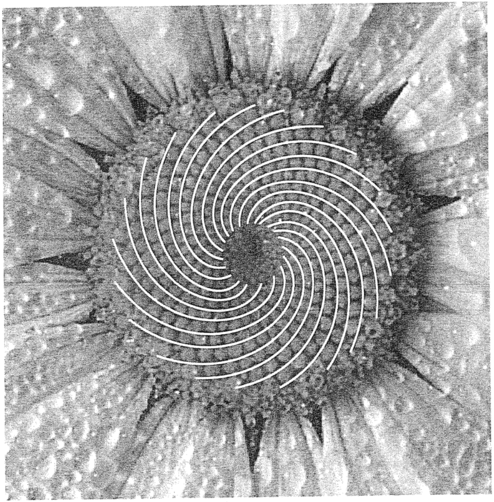
Fibonacci & The Daisy

(Reproduced from: 2005 Key College Publishing, *Instructor Resources and Adjunct Guide: The Heart of Mathematics*, Burger/Starbird/Bergstrand)



Fibonacci & The Daisy

(Reproduced from: *ibid*)



21 spirals

Fibonacci & The Daisy

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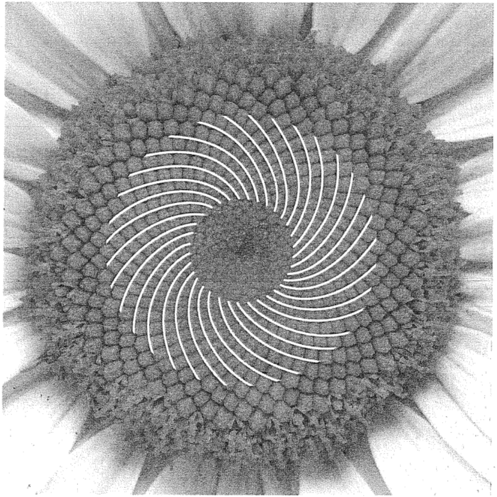

34 spirals

Fibonacci & Sunflower Spirals

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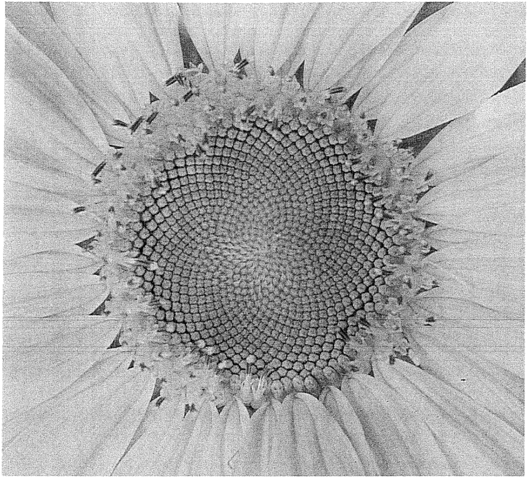
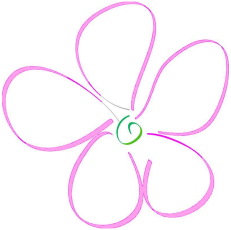


More Fibonacci Numbers And Nature

A great reference to check out is:

`http://www.maths.surrey.ac.uk/hosted-sites/R.Knott/Fibonacci/fibnat.html#section3`



QUESTIONS???