

THE UNIVERSITY OF MANITOBA

Dec 13, 2005

Paper #221

Departments & Course Numbers: 136.102/054.02

Examination: Math in Art

Examiners: Dr. R. Padmanabhan and Prof. T. Lysenko

Final Examination

Time 2 Hours

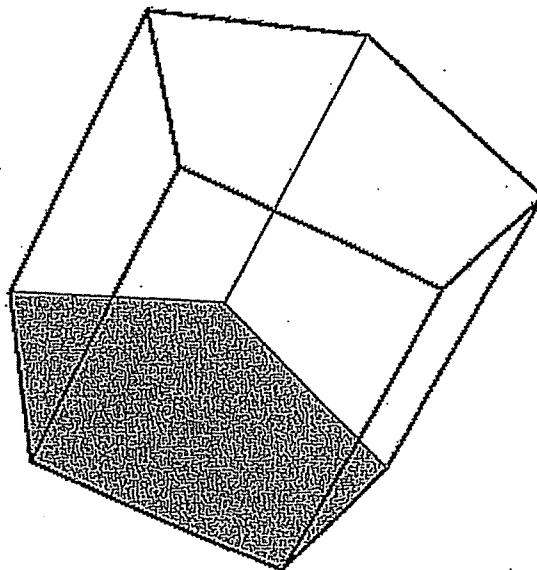
7. (a) Complete this table for the five Platonic solids.

Name of the solid	V = # of Vertices	E = # of Edges	F = # of Faces	# of Sides of each Face	# of Faces at each Vertex	V-E+F
Tetrahedron	4	6	4	3	3	2
Cube						
Octahedron						
Icosahedron						
Dodecahedron						

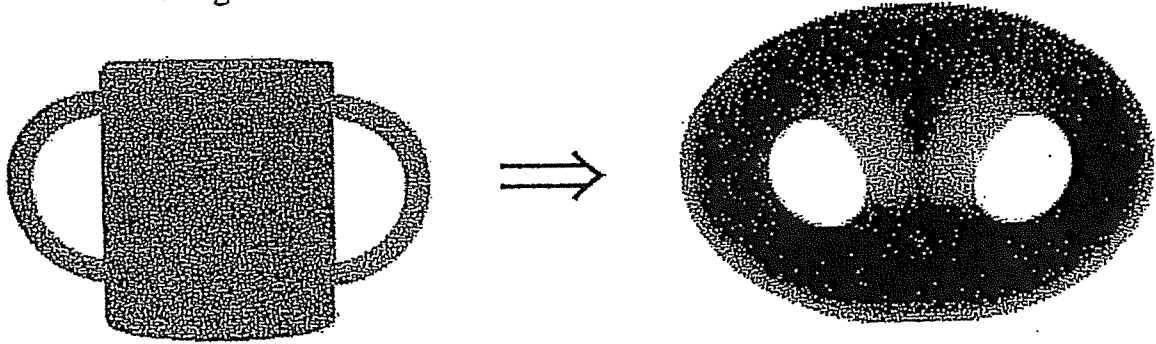
7 (b) State the Euler's theorem giving a formula connecting the vertices, edges and faces of a convex polyhedra.

(c) Calculate the values of V, E and F for the polyhedron shown below.

(d) What is the value of V-E+F here?



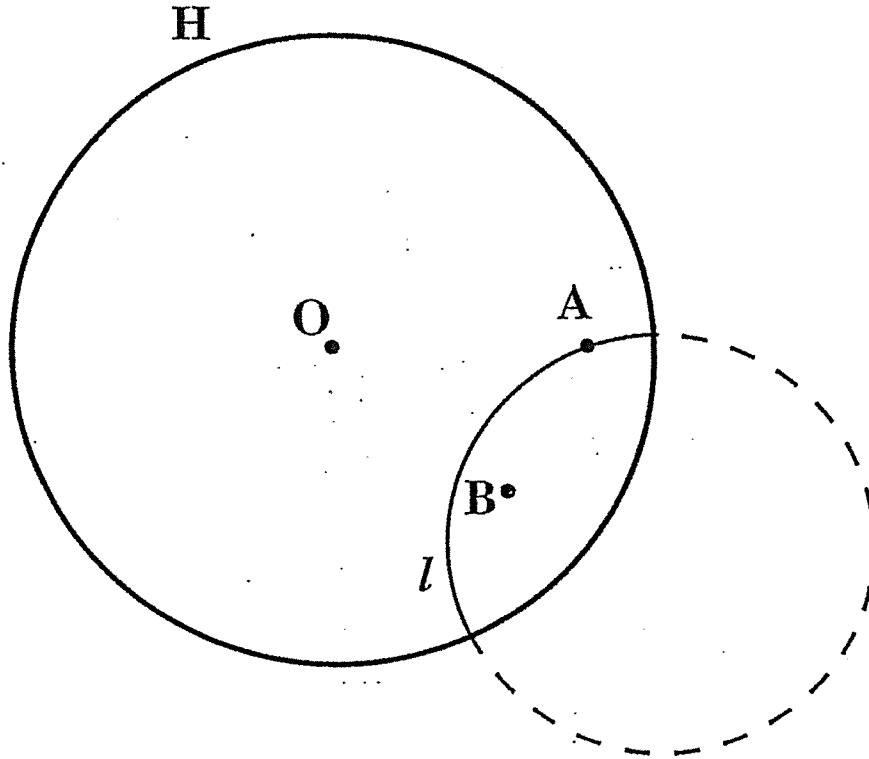
- 6 Apply a sequence of topological transformations (e.g. the continuous moulding of, say the ceramics or clay), to obtain the shape of a face mask shown on the right from the shape of the coffee mug with two handles as shown on the left. Show at least three or four in-between stages.



[8] 7. We are given a hyperbolic line l , a point A on that hyperbolic line, and a point B outside the line l .

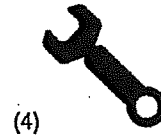
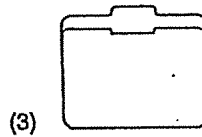
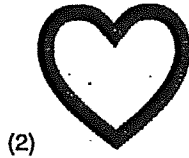
(a) Construct one hyperbolic line (label it h_1) parallel to l and passing through B , and one hyperbolic line (label it h_2) intersecting l and passing through B .

(b) Construct the hyperbolic line passing through both A and B .

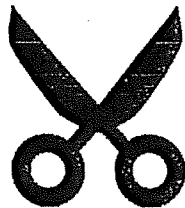


[7] 8. The objects depicted below consist of the black coloured points only.

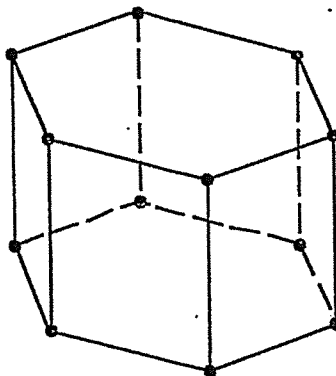
(a) Which of the following four designs are mutually homotopic?



(b) Show that the two designs shown below are homotopic by drawing at least three in-between sketches showing the left object can be continuously deformed into the right object.

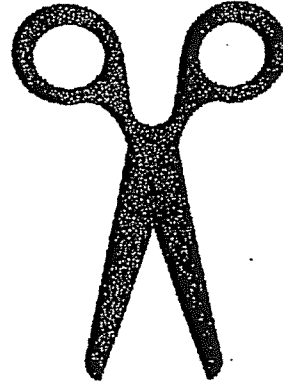
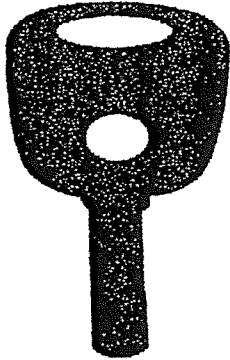


1. (a) State the Euler's formula connecting V -the number of vertices, E -the number of edges and F -the number of faces of a convex polyhedron (no proof is required).
- (b) Find the numbers V , E and F for the hexagonal prism given below. Calculate the value of $V - E + F$ for this convex polyhedron.



- (c) If a convex polyhedron has 10 vertices and 7 faces, find the number of edges of the polyhedron. Draw such an example of a polyhedron in the space given below.

2. Demonstrate that the two images given below are homotopic by drawing at least three in-between images showing how the key can be continuously deformed into the scissors.

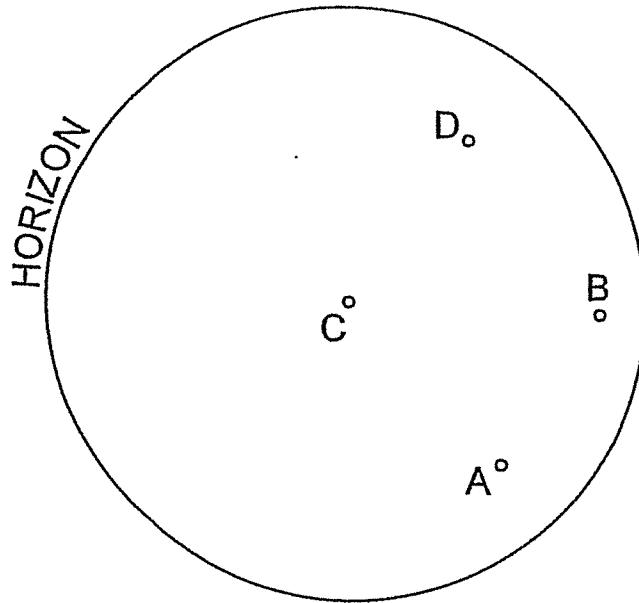


7. (a) Give the Euler's formula connecting V , E and F for a convex polyhedron
(recall V = # of vertices, E = # of edges and F = # of faces).
- (b) Let P be the convex polyhedron obtained by glueing together two identical regular tetrahedra along one face (to obtain a triangular dipyramid). Find the values of V , E and F for this polyhedron. Calculate the value of $V-E+F$.
- (c) Is this a regular polyhedron? Give reasons for your answer.

8. Hyperbolic Plane Constructions.

- (a) Draw the hyperbolic line joining the two points C and D.
- (b) Draw the hyperbolic line joining the two points A and B.
- (c) Draw two hyperbolic lines through the point D and parallel to AB.
- (d) How many such lines can be drawn through D and parallel to AB?

Note that the point C is the centre of the horizon circle.

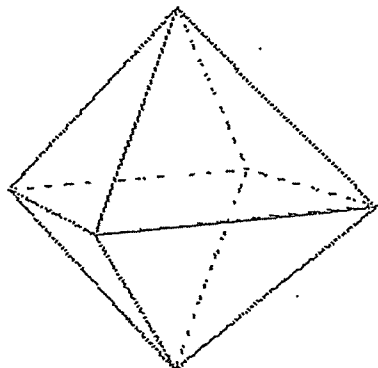


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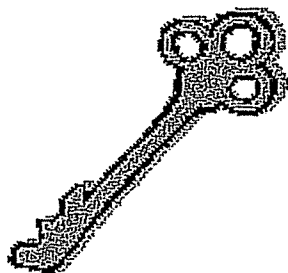
April 20, 2005
PAPER NO: 402
DEPARTMENT & COURSE NO: 136.102/054.102
EXAMINATION: Math in Art
EXAMINERS: S. Kalajdziewski, R. Padmanabhan

FINAL EXAMINATION
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9. (a) Compute the Euler characteristic of the octahedron (see the picture).



- (b) What is the genus of the surface of the key depicted below? (Note that the key is not flat – it is a 3 dimensional object).



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DATE: December 12, 2009
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FINAL EXAMINATION
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 TIME: 2 hours
 EXAMINER: M. Davidson

- [8] 4. (a) Fill in the following table with the appropriate information about the Platonic solids.

Name of Solid	number of Faces	number of Edges	number of Vertices
	12		20
cube		12	
icosahedron		30	
tetrahedron			4
	8	12	

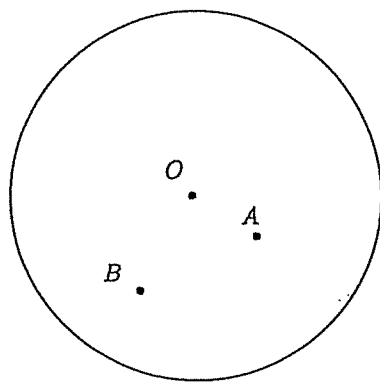
- (b) The icosododecahedron is an Archimedean solid that has 12 pentagon faces and 20 triangle faces. How many edges and how many vertices does a icosododecahedron have?
 (Hint: there is enough information given to calculate the number of edges. Use the Euler characteristic to find the number of vertices)

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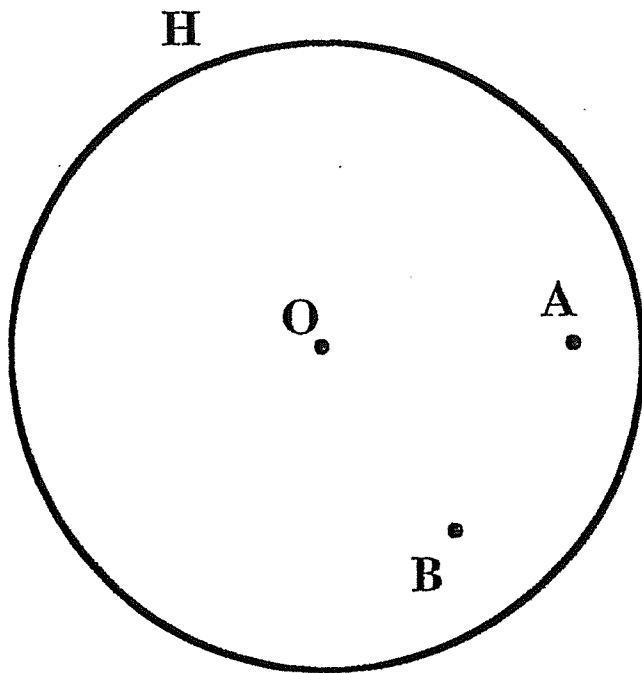
DATE: December 12, 2009
PAPER # 182\183
COURSE: MATH\FA 1020
EXAMINATION: Math in Art

FINAL EXAMINATION
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TIME: 2 hours
EXAMINER: M. Davidson

- [9] 7. Below is a hyperbolic plane, having center O .
- Construct the hyperbolic line, labeled h_1 , that passes through the points A and B .
 - Construct one hyperbolic line that intersects h_1 , label it h_2 , and one hyperbolic line that is parallel to h_1 , label it h_3 .

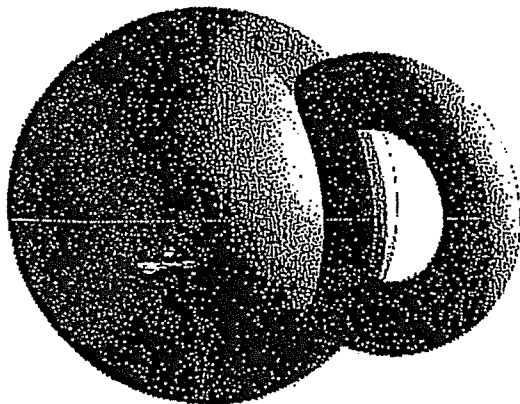


7. Construct the hyperbolic triangle with vertices A , B and O as shown below, where O is the center of the horizon circle H .



8. (a) Compute the genus of the three-dimensional surface depicted below (a sphere with a handle).

(b) Find the Euler characteristic of the object shown below; use part (a) to justify your answer.



3. Construct a **hyperbolic square** with vertices at A , B , C , and D . [You may use the fact that the four vertices are equidistant from the center O of the Poincaré model we are using, and that the distances between every two consecutive vertices of the square are the same.]

