

UNIVERSITY OF MANITOBA

DATE: February 21, 2006

MIDTERM

PAGE: 4 of 4

DEPARTMENT & COURSE NO: 136.102

TIME: 75 minutes

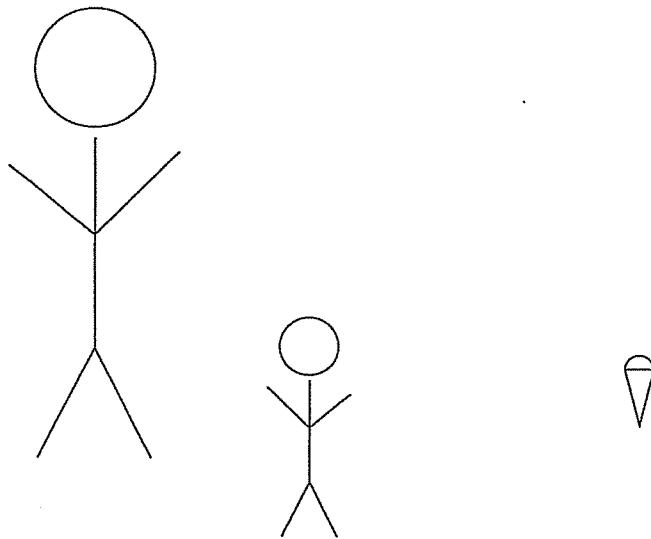
EXAMINATION: Math in Art

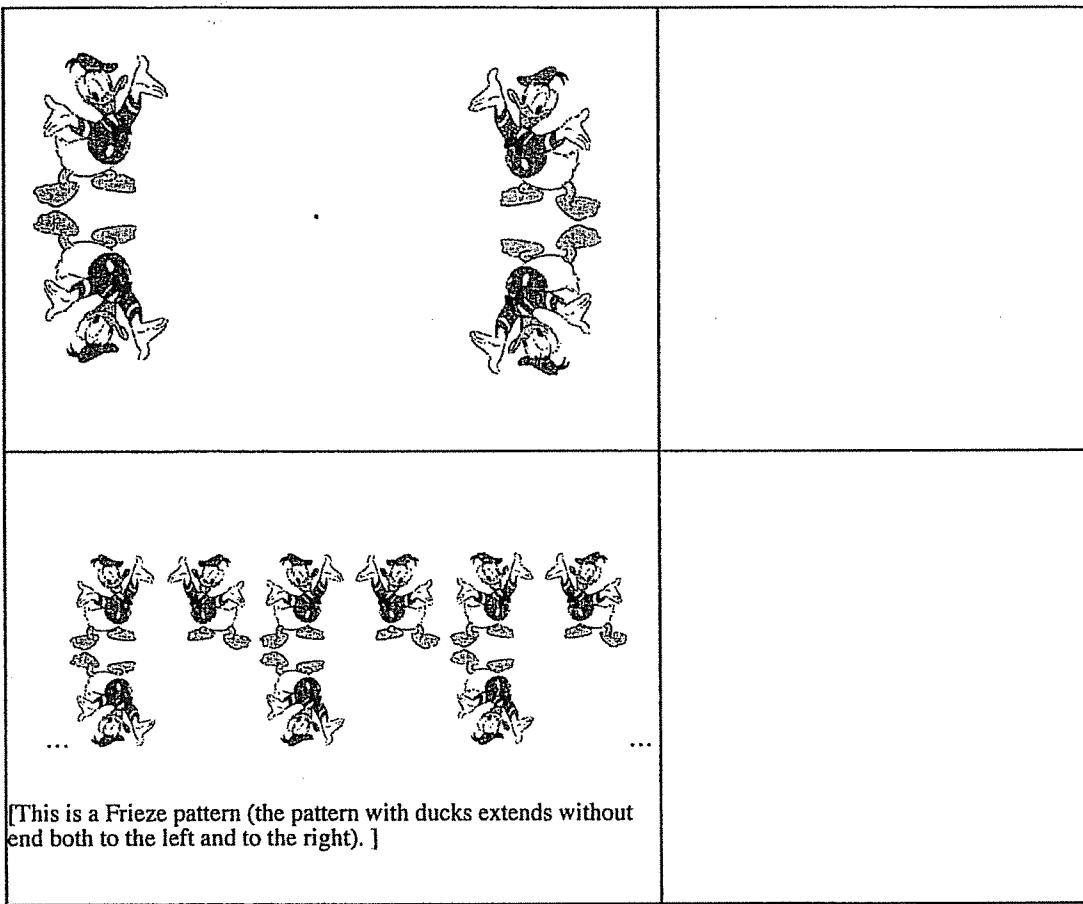
EXAMINER: M. Davidson

[9] 4. (a) What are the Fibonacci numbers? (Give a definition)

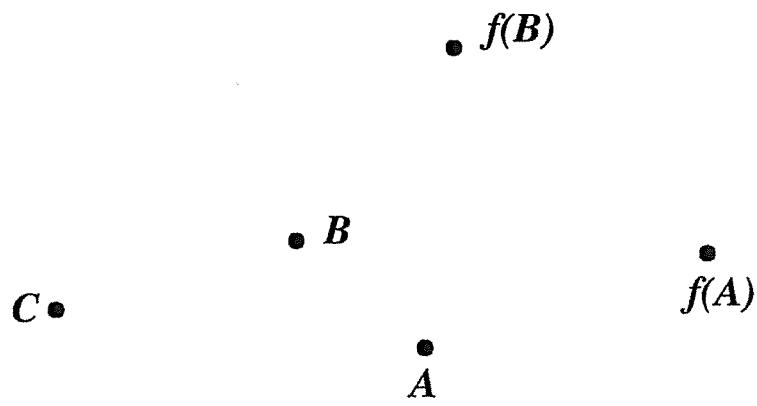
(b) Given that  $f_{24} = f_{23} + 17711$  and  $f_{21} = 10946$ , find  $f_{23}$ . (Here,  $f_n$  denotes the n-th Fibonacci number.)

[10] 5. In the following picture, the larger man (stick-figure) is the image of the smaller man under a central symmetry with a dialating factor of 2. Find (construct) the center of the central symmetry and sketch the image of the ice cream cone (by constructing the image of at least two points on the cone)





5. Suppose the point  $f(A)$  is the image of the point  $A$  and the point  $f(B)$  is the image of the point  $B$  under the central symmetry  $f$ . Find (construct) the center of the central symmetry  $f$  and then construct the image  $f(C)$  of the point  $C$  (as shown in the illustration) under the central symmetry  $f$ .



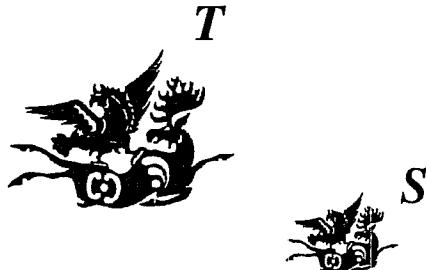
[10 points] 5. We know that the point  $f(A)$  is obtained from the point  $A$  by applying to it a central similarity  $f$  of stretching factor  $\alpha = \frac{1}{2}$ .

- (a) Construct the center  $O$  of the central similarity  $f$ .
- (b) Construct the image of the line segment  $CB$  under the central similarity  $f$ .

$C$  —————  $B$

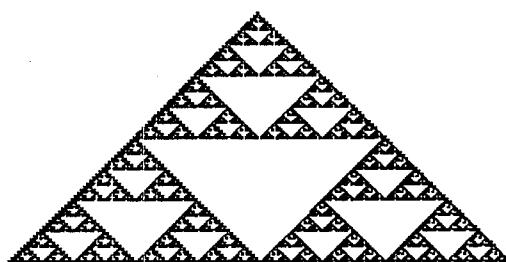
$f(A)$   $A$

5. (a) The object  $T$  is the image of the object  $S$  under an unknown central similarity  $f$ . Find the center of the central similarity  $f$ , and construct the image  $f(A)$  of the point  $A$  under the central similarity  $f$ .  
(We are again using a tattoo of the ancient mummy from Exercise 4.)



$A$   $\circ$

(b) The objects shown below is a fractal called Sierpinski triangle. As is visible from the graphics, the Sierpinski triangle is obtained by removing the triangle obtained by connecting the midpoints of the sides of the largest triangle, and then repeating that procedure ad infinitum (infinitely many times) to the smaller triangles we get in each step. Describe one central similarity of stretching factor not equal to 1, which moves the points of the Sierpinski triangle within itself. (In order to describe a central similarity you would need to identify its center and its stretching factor.)



# FA/MATH 102- Mathematics in Art

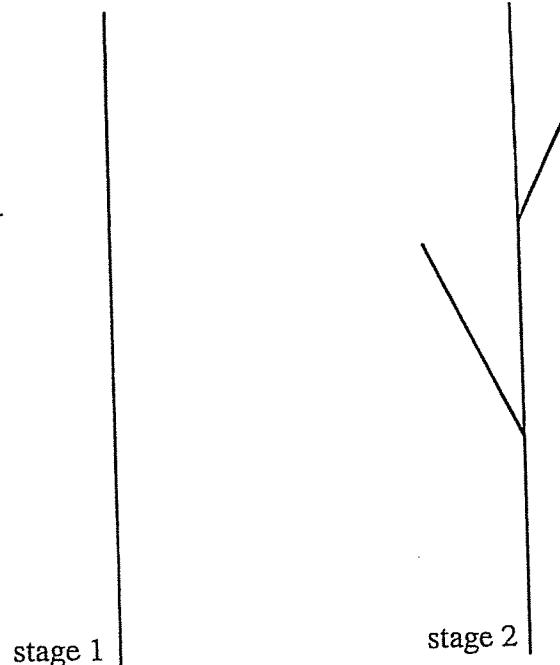
## Mid-Term Test, 19 October, 2006

page 3 of 5  
time 70 minutes

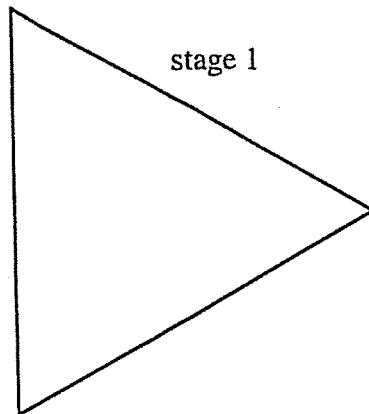
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4. Study the algorithm given by the first three stages of a fractal tree shown below and draw the next two stages of the fractal. Demonstrate self-similarity by circling a portion of your final sketch which is similar the entire tree in the previous stage.

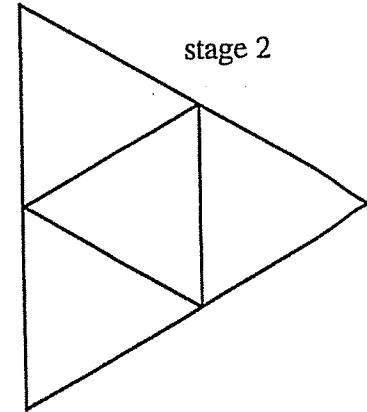
10



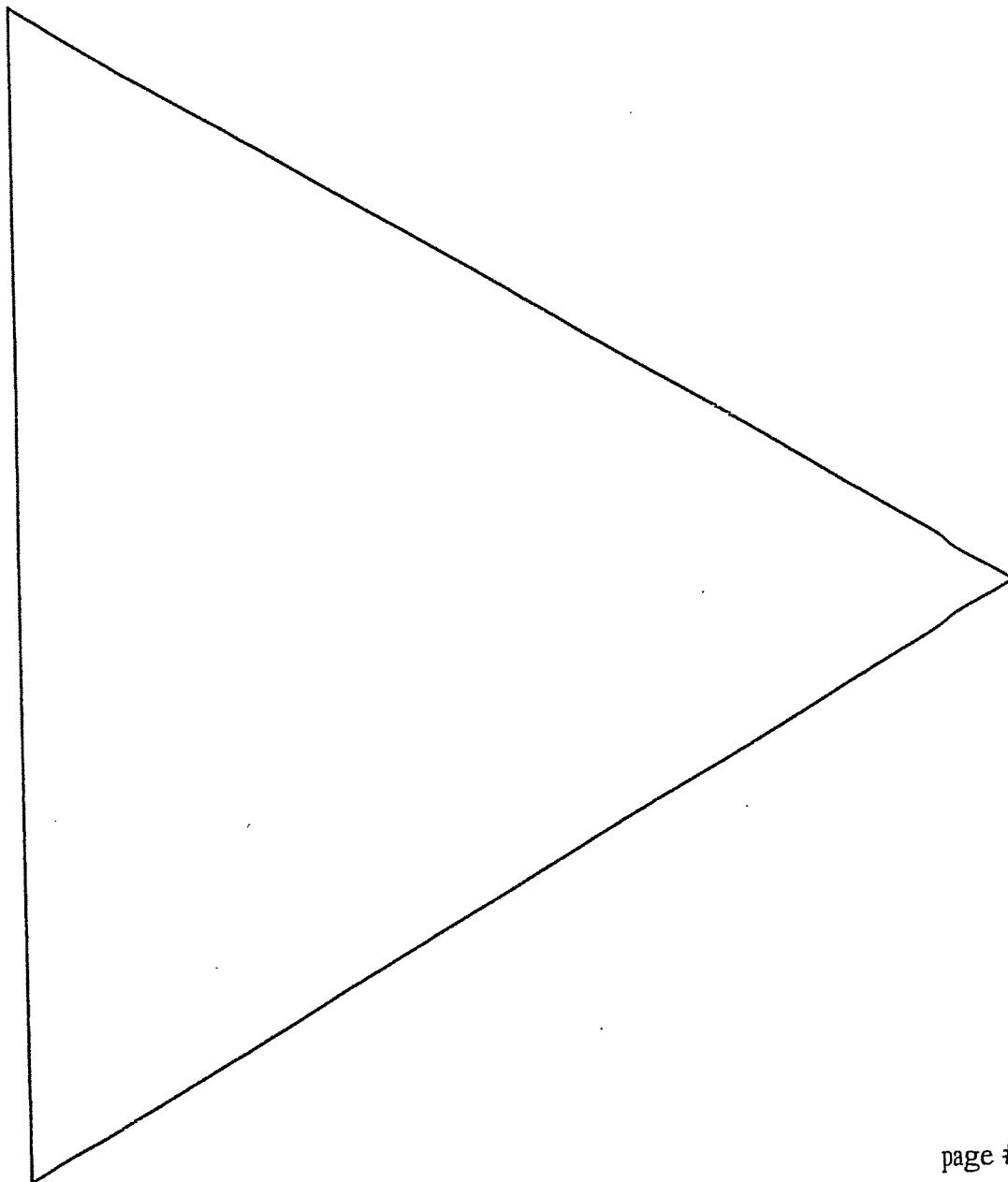
4. Following the algorithm for drawing a Sierpinski triangle suggested by the first two stages given here, draw next two stages (of the fractal) in the big triangle given below. Demonstrate the principle of self-similarity of the fractal by circling a portion of your final artwork which is similar to the whole design. (In making the design, you may shade those regions that are to be removed)



stage 1



stage 2



FA/MATH 1020 Mathematics in Art  
Mid-Term Exam, 28 February 2008.

7. Study the algorithm given by the first two stages of a fractal design given below and draw the next two stages of the fractal. Demonstrate the self-similarity property of the fractal by circling a portion of your final sketch that is similar to the entire design of the *previous* stage.



[7] 6. In the two figures below (Figure 1 and Figure 2) we show the first two steps in the construction of a fractal.

(a) Draw the figure representing the next step in the construction of the fractal. (The dot in the middle of the large circle in Figure 1 represents the center of that circle. You do NOT need to precisely construct the circles and the lines in the next step.)

(b) The final fractal  $F$  will be constructed after infinitely many steps (the first few of them are described in Figures 1, 2 and in the correct solution to question (a) here). Find a central similarity of stretching factor not equal to 1 that will send the fractal  $F$  into itself. (To get full marks here, you need to indicate in the figure you draw in part (a) where the center of the central similarity is, and you need to state a specific number for the stretching factor of that central similarity.)

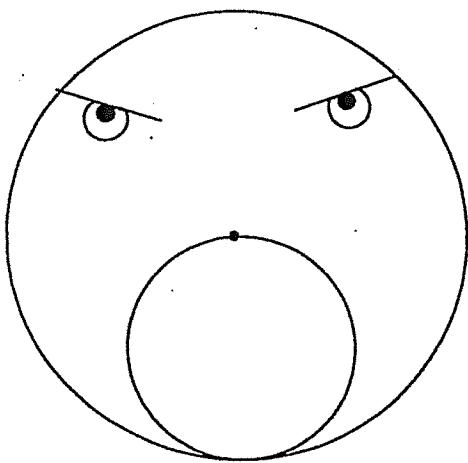


Figure 1.

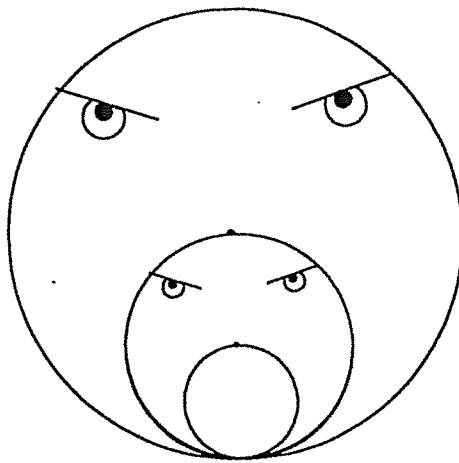


Figure 2.