MATH 1010, Summer 2018 **Dr. S. Cooper**

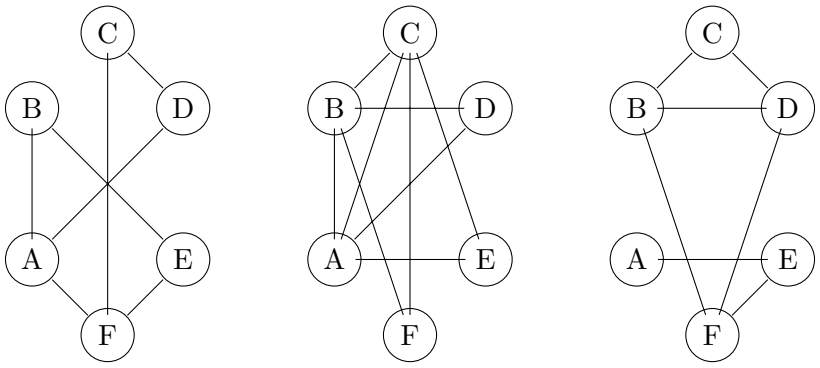
Tutorial Worksheet #6
Tuesday, June 19

Name: _____

Student Number: _____

Write your solutions to the following exercises on the provided paper. *Show all of your work.* Remember to use good notation and full sentences.

1. Do the following graphs have an Euler circuit? An Euler path? Neither? Write down an Euler circuit/path for those that do.



2. Draw a simple graph, if possible, with 6 vertices and 9 edges so that there are no circuits with exactly 3 edges.

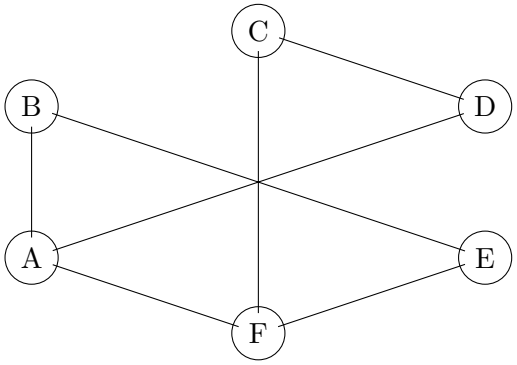
3. A simple graph with 12 edges has three vertices of degree 4, two vertices of degree 3, and the rest have degree 2. You showed via Worksheet #5 that such a graph has 8 vertices. Does such a graph have an Euler path or Euler circuit?

4. A clever kitten likes to get dinner from 4 different families including her own. The following table shows the distance (in metres) between the kitten's home and the other 3 families.

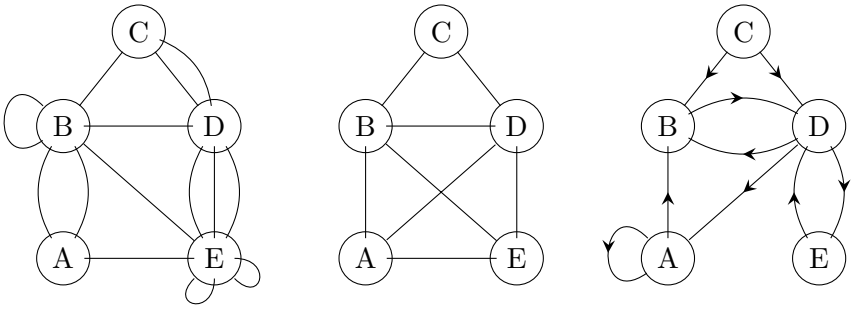
	Family 1	Family 2	Family 3
Home	500	700	200
Family 1		400	750
Family 2			300

In order to minimize the distance she travels, and supposing she starts and finishes at home, use the root finding method for Hamiltonian circuits to decide what order should the kitten visit each family?

5. Write down a Hamiltonian circuit and an Euler path of the graph below.



6. Write down the adjacency matrices for the following graphs/digraphs:



   For the first two, are the associated graphs simple?

7. For each graph/digraph in Exercise #6, calculate:

   (a) the number of routes of length 2 from vertex C to vertex A;

   (b) the number of routes of length 2 or less from vertex B to vertex D.

8. Draw the graph whose adjacency matrix is

$$\begin{bmatrix} 0 & 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 \end{bmatrix}.$$

   Does this graph have

   (a) an Euler circuit?

   (b) an Euler path?

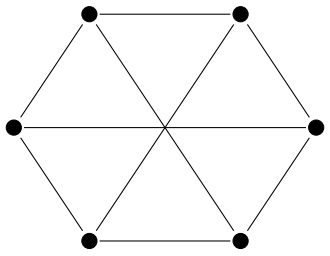
   Is there a way to obtain this information without drawing the graph?

9. Draw the digraph whose adjacency matrix is

$$\begin{bmatrix} 1 & 0 & 1 & 0 & 1 \\ 2 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 2 & 1 \\ 2 & 2 & 1 & 0 & 0 \end{bmatrix}.$$

Brief Answers:

1. The first graph has an Euler path: (A, B, E, F, C, D, A, F). The second graph has an Euler circuit (and hence path): $(C, B, A, C, E, A, D, B, F, C)$. The third graph has neither.

2. 

3. It has an Euler path since it has exactly two vertices of odd degree. It does not have an Euler circuit since it has vertices with odd degree.

4. The kitten has two options to travel a minimum distance (of 1400m):

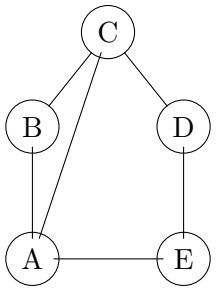
   - Home → Family 1 → Family 2 → Family 3 → Home
   - Home → Family 3 → Family 2 → Family 1 → Home

5. A Hamiltonian circuit is: (D, C, F, E, B, A, D); an Euler path is: (A, D, C, F, E, B, A, F)

6. $$\begin{bmatrix} 0 & 2 & 0 & 0 & 1 \\ 2 & 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 2 & 0 \\ 0 & 1 & 2 & 0 & 3 \\ 1 & 1 & 0 & 3 & 2 \end{bmatrix} \quad \text{and} \quad \begin{bmatrix} 0 & 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 & 1 \\ 1 & 1 & 0 & 1 & 0 \end{bmatrix} \quad \text{and} \quad \begin{bmatrix} 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

   The first graph is not simple and the second graph is simple.

7. (a) For the first graph: 2; for the second graph: 2; for the third graph: 1
   (b) For the first graph: 7; for the second graph: 4; for the third graph: 1



8. The graph has an Euler path but not an Euler circuit. It is difficult to tell from the adjacency matrix alone because it can be hard to tell from the adjacency matrix whether or not a graph is connected.

9. We have