MATH 1010, Summer 2018 **Dr. S. Cooper**

Tutorial Worksheet #5
Tuesday, June 12

Name: ______________________________

Student Number: ______________________________

Write your solutions to the following exercises on the provided paper. *Show all of your work.* Remember to use good notation and full sentences.

1. Find the inverse of the following matrices, if possible.

   (a) $A = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 0 & 3 \\ 4 & -3 & 8 \end{bmatrix}$

   (b) $A = \begin{bmatrix} 1 & -2 & -1 \\ -1 & 5 & 6 \\ 5 & -4 & 5 \end{bmatrix}$

   (c) $A = \begin{bmatrix} 1 & 0 & -2 \\ -3 & 1 & 4 \\ 2 & -3 & 4 \end{bmatrix}$

   (d) $A = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 \end{bmatrix}$

   (e) $A = \begin{bmatrix} 1 & 2 & -1 \\ -4 & -7 & 3 \\ -2 & -6 & 4 \end{bmatrix}$

2. Let $A = \begin{bmatrix} -1 & -7 & -3 \\ 2 & 15 & 6 \\ 1 & 3 & 2 \end{bmatrix}$. Find the third column of A^{-1} without computing the other columns.

3. Let $A = \begin{bmatrix} 3 & 2 \\ 8 & 5 \end{bmatrix}$.

   (a) Find A^{-1}.
   (b) Find A^T.
   (c) Find $(A^T)^{-1}$.
   (d) Find $(A^{-1})^T$.

4. For each of the following systems, use a matrix inverse to find the solution.

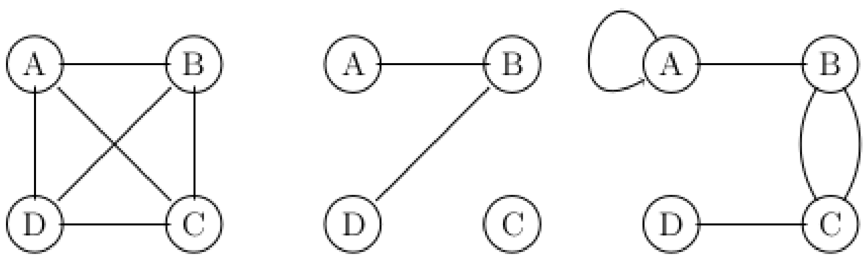
   (a)

$$\begin{aligned} 3x + 4y &= 3 \\ 5x + 6y &= 7 \end{aligned}$$

   (b)

$$\begin{aligned} 2x + 3y + z &= 4 \\ 3x + 3y + z &= 8 \\ 2x + 4y + z &= 5 \end{aligned}$$

5. Which of the following graphs are simple?



6. Draw all non-equivalent simple graphs on 5 vertices with four edges.

7. Below are the proposed degrees of the vertices in a simple graph. Determine whether such a graph exists. Give reasons if one does not exist, and draw a simple, planar graph when one does exist.

   (a) $\{2, 3, 4, 3, 2, 5\}$
   (b) $\{3, 3, 2, 2, 4\}$
   (c) $\{0, 2, 3, 1, 4\}$

8. A simple graph with 12 edges has three vertices of degree 4, two vertices of degree 3, and the rest have degree 2. How many vertices does the graph have?

Brief Answers:

1. (a) $A^{-1} = \begin{bmatrix} -9/2 & 7 & -3/2 \\ -2 & 4 & -1 \\ 3/2 & -2 & 1/2 \end{bmatrix}$

   (b) A^{-1} does not exist

   (c) $A^{-1} = \begin{bmatrix} 8 & 3 & 1 \\ 10 & 4 & 1 \\ 7/2 & 3/2 & 1/2 \end{bmatrix}$

   (d) $A^{-1} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & -1 & 1 \end{bmatrix}$

   (e) A^{-1} does not exist

2. $\begin{bmatrix} 3 \\ 0 \\ -1 \end{bmatrix}$. To find this you row reduce the matrix $\left[\begin{array}{ccc|c} -1 & -7 & -3 & 0 \\ 2 & 15 & 6 & 0 \\ 1 & 3 & 2 & 1 \end{array}\right]$.

3. (a) $A^{-1} = \begin{bmatrix} -5 & 2 \\ 8 & -3 \end{bmatrix}$

   (b) $A^T = \begin{bmatrix} 3 & 8 \\ 2 & 5 \end{bmatrix}$

   (c) $(A^T)^{-1} = \begin{bmatrix} -5 & 8 \\ 2 & -3 \end{bmatrix}$

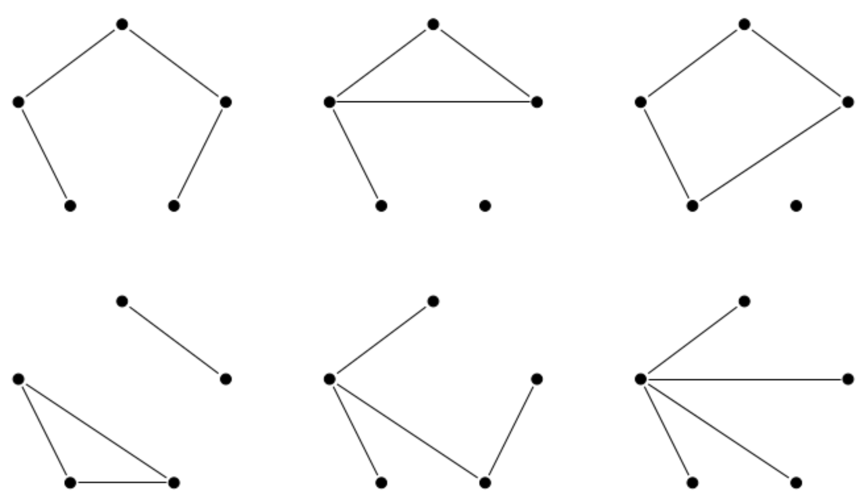
   (d) $(A^{-1})^T = \begin{bmatrix} -5 & 8 \\ 2 & -3 \end{bmatrix}$

4. (a) $x = 5, y = -3$

   (b) $x = 4, y = 1, z = -7$

5. The first two graphs are simple.

6. There are 6 graphs:



7. (a) No such graph can exist (the sum of the proposed degrees is odd).
   (b) Such a graph exists - there are many examples.
   (c) No such graph exists (the largest possible degree for the non-isolated vertices is 3 but one proposed degree is 4).

8. 8