Tutorial Worksheet #4
Tuesday, June 5

Name: _____

Student Number: _____

Write your solutions to the following exercises on the provided paper. *Show all of your work.* Remember to use good notation and full sentences.

1. Consider the matrices

$$A = \begin{bmatrix} 2 & 2 & -1 \\ -1 & 3 & 0 \\ 0 & 7 & 4 \end{bmatrix}, B = \begin{bmatrix} -1 & -2 & 3 \\ -2 & 4 & 1 \\ 3 & 1 & 3 \end{bmatrix}, C = \begin{bmatrix} 1 & 1 \\ 2 & 0 \\ 0 & -3 \end{bmatrix}, D = \begin{bmatrix} 4 & 3 & -3 \\ 0 & -2 & 1 \end{bmatrix}.$$

   Find (if possible):

   (a) BA, AC, CB, DC and CD

   (b) $A(DB)^T$

2. Consider the matrices

$$A = \begin{bmatrix} 2 & 2 & -1 & 0 & 1 & 8 \\ -1 & 3 & 0 & -1 & -2 & 0 \\ 0 & 7 & 4 & 7 & 3 & -1 \\ 11 & -3 & 1 & 11 & 12 & 10 \\ 10 & 2 & -4 & -7 & 5 & 1 \end{bmatrix} \text{ and } B = \begin{bmatrix} 1 & 2 & 1 & 10 & 3 & 8 \\ 1 & -3 & 4 & 11 & 2 & 2 \\ 0 & 0 & 4 & 3 & -3 & -10 \\ 6 & 3 & 1 & -1 & 1 & 0 \\ 10 & 6 & 4 & -7 & 4 & 1 \\ 6 & 2 & -4 & -6 & 5 & 1 \end{bmatrix}.$$

   Find the following, if they exist:

   (a) the $(5, 4)$ entry of AB

   (b) the $(2, 3)$ entry of BA

   (c) the $(5, 1)$ entry of BA^T

   (d) the $(3, 6)$ entry of AB^T

3. Solve the following systems using Gaussian elimination:

   (a)

$$3x - y + 3z = 0$$
$$4x - 2y + 5z = -2$$
$$7x - y + 2z = 3$$

(b)

$$2x - y + 4z + w = -13$$
$$x + y - 2z + 3w = 11$$
$$x - 3y + z = -9$$
$$4y + 2z - w = 1$$

4. A rich and famous actor has 18 vehicles: cars (C), trucks (T) and motorbikes (M). He has 4 more cars than trucks and motorbikes combined. Suppose the value of each car is 90K, each truck is 70K and each motorbike is 40K, and that the combined value of all his vehicles is $1,390,000. How many of each vehicle does he own?

   Hint: Form a system of equations from the given information and use Gaussian elimination to solve the system.

5. Use Gauss-Jordan elimination to find the RREF of the augmented matrix of the following system of equations:

$$x - 2y + 3z = 7$$
$$-3x + 5y + 5z = 7$$
$$4x - 7y - 2z = 1$$

   How many solutions does the system have? If it has a unique solution, find it. If it has infinitely many solutions, state them in terms of t.

6. Find the reduced row echelon form of the augmented matrix of the following system of equations:

$$-2x - 3y + z = 0$$
$$x + 2y - z = 4$$
$$-5x - 7y + 2z = 4$$

   How many solutions does the system have? If it has a unique solution, find it. If it has infinitely many solutions, state them in terms of t.

7. Suppose that the augmented matrix for a linear system of equations has been reduced to

$$\left[\begin{array}{ccc|c} 1 & 0 & -4 & 3 \\ 0 & 1 & -3 & 2 \\ 0 & 0 & 0 & a \end{array}\right]$$

   For what values of a does the system have:

   (a) infinitely many solutions?
   (b) no solutions?
   (c) a unique solution?

Brief Answers:

1. We have

   (a) • $BA = \begin{bmatrix} 0 & 13 & 13 \\ -8 & 15 & 6 \\ 5 & 30 & 9 \end{bmatrix}$

   • $AC = \begin{bmatrix} 6 & 5 \\ 5 & -1 \\ 14 & -12 \end{bmatrix}$

   • CB is not defined

   • $DC = \begin{bmatrix} 10 & 13 \\ -4 & -3 \end{bmatrix}$

   • $CD = \begin{bmatrix} 4 & 1 & -2 \\ 8 & 6 & -6 \\ 0 & 6 & -3 \end{bmatrix}$

   (b) $A(DB)^T = \begin{bmatrix} -42 & -1 \\ 22 & -28 \\ 31 & -45 \end{bmatrix}$

2. (a) 76

   (b) BA is not defined

   (c) 40

   (d) -30

3. (a) $x = \dfrac{5}{6}, y = \dfrac{7}{2}, z = \dfrac{1}{3}$

   (b) $x = 0, y = 2, z = -3, w = 1$

4. 11 cars, 4 trucks, 3 motorbikes

5. No solutions

6. Infinitely many solutions:

$$x = -12 - t$$
$$y = 8 + t$$
$$z = t$$

7. (a) $a = 0$

   (b) All real numbers a such that $a \neq 0$

   (c) No value of a yields a unique solution