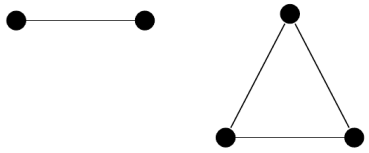
MATH 1010, Summer 2018 **Dr. S. Cooper**

Quiz 6 Solutions

Name and Student Number: _____

Write your solutions to the following exercises in the space provided. *Show all of your work.* Remember to use good notation and full sentences. *Good Luck!*

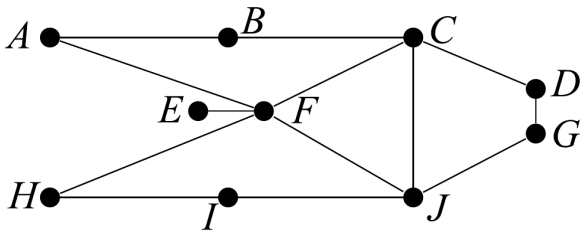
1. Consider the following graph:



Is the graph connected? [1 pt]

Solution: The graph is *not* connected.

2. Consider the following graph:



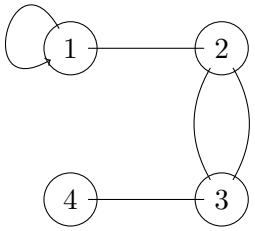
(a) Does this graph have an Euler circuit? If so, find one. If not, explain why. [4 pts]

Solution: By Euler's First Theorem, the graph does not have an Euler circuit since not every vertex has even degree (for example, vertex F has degree 5).

(b) Does this graph have an Euler path? If so, find one. If not, explain why. [3 pts]

Solution: The graph does have an Euler path since it has exactly two vertices (namely, E and F) of odd degree. One Euler path is: $(E, F, A, B, C, D, G, J, F, C, J, I, H, F)$.

3. Write down the adjacency matrix for the following graph: [4 pts]



Solution:

$$A = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & 0 & 2 & 0 \\ 0 & 2 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}.$$

4. Below is the adjacency matrix (A) of a graph along with A^2:

$$A = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 \end{bmatrix} \quad \text{and} \quad A^2 = \begin{bmatrix} 2 & 1 & 1 & 0 & 2 & 1 \\ 1 & 2 & 0 & 0 & 2 & 1 \\ 1 & 0 & 4 & 2 & 1 & 2 \\ 0 & 0 & 2 & 2 & 0 & 1 \\ 2 & 2 & 1 & 0 & 3 & 1 \\ 1 & 1 & 2 & 1 & 1 & 3 \end{bmatrix}.$$

Without drawing the graph, determine the following:

(a) How many vertices does the graph have? [1 pt]

Solution: 6

(b) Is the graph simple? [1 pt]

Solution: Yes since there are only 0's along the diagonal (i.e., the graph has no loops) and the non-diagonal entries are all 0 or 1 (i.e., the graph does not have parallel edges).

(c) What is the degree of vertex 4? [1 pt]

Solution: 2 (the row sum of row 4 = the column sum of column 4)

(d) What is the number of routes of length 2 joining vertex 3 to vertex 4? [1 pt]

Solution: 2 since 2 is the $(3, 4)$ entry of A^2

(e) What is the number of routes of length 2 joining vertex 6 to itself? [1 pt]

Solution: 3 since 3 is the $(6, 6)$ entry of A^2

(f) What is the number of routes of length at most 2 joining vertex 6 to vertex 3? [1 pt]

Solution: $1 + 2 = 3$ which is the $(6, 3)$ entry of $A + A^2$

Exercise (5) On Next Page ...

5. The following matrix A is the adjacency matrix for a digraph:

$$\begin{bmatrix} 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 \end{bmatrix}.$$

Without drawing the digraph, determine the following:

(a) What is the number of edges directed away from vertex 3? [1 pt]

Solution: 3 since the row sum of the third row is $1 + 1 + 0 + 1 = 3$

(b) What is the number of edges directed toward vertex 4? [1 pt]

Solution: 2 since the column sum of column 4 is $1 + 0 + 1 + 0 = 2$