MATH 1010, Summer 2018 **Dr. S. Cooper**

Quiz 5 Solutions

Name and Student Number: ____________________

Write your solutions to the following exercises in the space provided. *Show all of your work.* Remember to use good notation and full sentences. *Good Luck!*

1. Find the inverse of the matrix
$$A = \begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix}.$$
[6 pts]

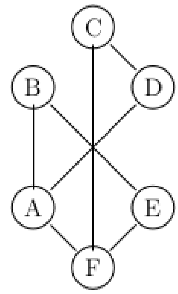
Solution: We have
$$\left[\begin{array}{cc|cc} 2 & 5 & 1 & 0 \\ 1 & 3 & 0 & 1 \end{array}\right] \xrightarrow[R_1 \leftrightarrow R_2]{} \left[\begin{array}{cc|cc} 1 & 3 & 0 & 1 \\ 2 & 5 & 1 & 0 \end{array}\right] \xrightarrow[R_2 \to R_2 - 2R_1]{} \left[\begin{array}{cc|cc} 1 & 3 & 0 & 1 \\ 0 & -1 & 1 & -2 \end{array}\right]$$
$$\xrightarrow[R_2 \to -R_2]{} \left[\begin{array}{cc|cc} 1 & 3 & 0 & 1 \\ 0 & 1 & -1 & 2 \end{array}\right] \xrightarrow[R_1 \to R_1 - 3R_2]{} \left[\begin{array}{cc|cc} 1 & 0 & 3 & -5 \\ 0 & 1 & -1 & 2 \end{array}\right]$$
and so
$$A^{-1} = \begin{bmatrix} 3 & -5 \\ -1 & 2 \end{bmatrix}.$$

2. Let $A = \begin{bmatrix} 1 & 4 \\ 2 & 9 \end{bmatrix}$. Use the fact (without verification) that $A^{-1} = \begin{bmatrix} 9 & -4 \\ -2 & 1 \end{bmatrix}$ to solve the following system of linear equations:
$$x + 4y = 3$$
$$2x + 9y = -1$$
No marks will be awarded for using any other method. [4 pts]

Solution:
$$\begin{bmatrix} x \\ y \end{bmatrix} = A^{-1} \begin{bmatrix} 3 \\ -1 \end{bmatrix} = \begin{bmatrix} 9 & -4 \\ -2 & 1 \end{bmatrix} \begin{bmatrix} 3 \\ -1 \end{bmatrix} = \begin{bmatrix} 31 \\ -7 \end{bmatrix}$$
and so
$$x = 31, \quad \text{and} \quad y = -7.$$

Turn Over For Exercises (3) – (5) ...

3. Consider the graph below:



   (a) How many vertices are in the graph? [1 pt]

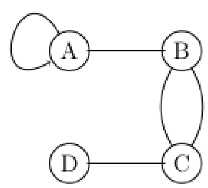
   Solution: 6

   (b) How many edges are in the graph? [1 pt]

   Solution: 7

4. Draw a graph which is not simple and which has degree set $\{3, 3, 3, 1\}$. [4 pts]

   Solution: There are a number of such graphs. Here is one example:



5. Is it possible to have a graph with 14 edges, 3 vertices of degree 4, 5 vertices of degree 1, and every other vertex have degree 3? Fully justify your answer. [4 pts]

   Solution: Let x denote the number of vertices of degree 3. Then, by the Handshaking Lemma,

$$\begin{aligned} 3(4) + 5(1) + 3x &= 2(14) \\ 12 + 5 + 3x &= 28 \\ 17 + 3x &= 28 \\ 3x &= 11 \\ x &= \frac{11}{3} \end{aligned}$$

   Thus, if such a graph exists then there would be $\frac{11}{3}$ vertices of degree 3 which is nonsense. Therefore, no such graph exists.