Quiz 4 Solutions

Name and Student Number: _____

Write your solutions to the following exercises in the space provided. *Show all of your work.* Remember to use good notation and full sentences. *Good Luck!*

1. Consider the matrices:

$$A = \begin{bmatrix} 1 & -2 & 3 \\ 0 & 2 & -1 \\ 5 & 2 & 3 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} 1 & 2 \\ 5 & -1 \\ 3 & 4 \end{bmatrix}.$$

   (a) Find AB or state that it is not defined.   [2 pts]

   Solution:

   $$AB = \begin{bmatrix} 0 & 16 \\ 7 & -6 \\ 24 & 20 \end{bmatrix}.$$

   (b) Find BA or state that it is not defined.   [2 pts]

   Solution: B has size 3×2 and A has size 3×3. Since the number of columns of B does not equal the number of rows of A, the product BA is not defined.

2. Which of the following matrices are in row echelon form (REF)? Reduced row echelon form (RREF)?   [4 pts]

$$A = \begin{bmatrix} 1 & 0 & -3 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 0 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix},$$

$$D = \begin{bmatrix} 1 & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}, \quad E = \begin{bmatrix} 1 & 2 & 1 & 3 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad F = \begin{bmatrix} 1 & 2 & 1 & 3 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & -1 \end{bmatrix}.$$

   Solution:

   Matrices in REF: <u>A, B, C, D</u>

   Matrices in RREF: <u>B, C</u>

3. You are given a system of linear equations and while performing Gauss-Jordan elimination you get the following augmented matrix:

$$\left[\begin{array}{cccc|c} 1 & -2 & 0 & 5 & 2 \\ 0 & 0 & 1 & -3 & 5 \\ 0 & 0 & 0 & 0 & 7 \end{array}\right]$$

How many solutions are there to the system? Justify your answer. [2]

Solution: There are no solutions since the last row says "$0 = 7$".

4. You are given a system of linear equations with variables x, y and z. **After** performing Gauss-Jordan elimination you end up with the following augmented matrix:

$$\left[\begin{array}{ccc|c} 1 & 0 & 4 & 3 \\ 0 & 1 & 5 & -2 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right].$$

Find all solutions to the system. [3]

Solution: $x = -4z + 3, y = -5z - 2$ and z arbitrary

5. You are given a system of linear equations with variables x, y and z. **After** performing Gaussian elimination you end up with the following augmented matrix:

$$\left[\begin{array}{ccc|c} 1 & 2 & 3 & 4 \\ 0 & 1 & 5 & 6 \\ 0 & 0 & 1 & 7 \end{array}\right].$$

Find all solutions to the system. Do not further reduce the augmented matrix. [4 pts]

Solution: We have

$$x + 2y + 3z = 4$$
$$y + 5z = 6$$
$$z = 7$$

So, back-substitution yields

$$y + 5(7) = 6 \implies y + 35 = 6 \implies y = 6 - 35 = -29$$

and

$$x + 2(-29) + 3(7) = 4 \implies x - 58 + 21 = 4 \implies x - 37 = 4 \implies x = 37 + 4 = 41.$$

Thus, there is one unique solution:

$$x = 41, \quad y = -29, \quad z = 7.$$

Exercise (6) On Next Page ...

6. Consider the system of linear equations:

$$2x - 3y + 3z = 6$$
$$x + 2y - z = 3$$
$$x - y + z = 2.$$

(a) What is the augmented matrix of the system? [1 pt]

Solution:
$$\left[\begin{array}{ccc|c} 2 & -3 & 3 & 6 \\ 1 & 2 & -1 & 3 \\ 1 & -1 & 1 & 2 \end{array}\right].$$

(b) While solving the system via Gaussian elimination, you obtain the matrix:

$$\left[\begin{array}{ccc|c} 1 & 2 & -1 & 3 \\ 0 & 1 & -1 & -2 \\ 0 & 3 & -2 & 1 \end{array}\right].$$

State and perform the next elementary row operation that you would use. (Do not completely solve the system.) [2 pts]

Solution: With Guassian elimination, you want to transform the augmented matrix in REF. Thus, you should perform the operation $R_3 \to R_3 - 3R_2$ which yields the matrix

$$\left[\begin{array}{ccc|c} 1 & 2 & -1 & 3 \\ 0 & 1 & -1 & -2 \\ 0 & 0 & 1 & 7 \end{array}\right].$$