

Quiz 3 Solutions

Name and Student Number: _____

Write your solutions to the following exercises in the space provided. *Show all of your work.* Remember to use good notation and full sentences. *Good Luck!*

1. Consider the inequality $-2y - 6x \geq -6$.

(a) Is the point $(0,0)$ a solution to this inequality? [1 pt]

Solution: $(0,0)$ is a solution since if we substitute $x = y = 0$ into the left-hand side we have

$$-2(0) - 6(0) = 0$$

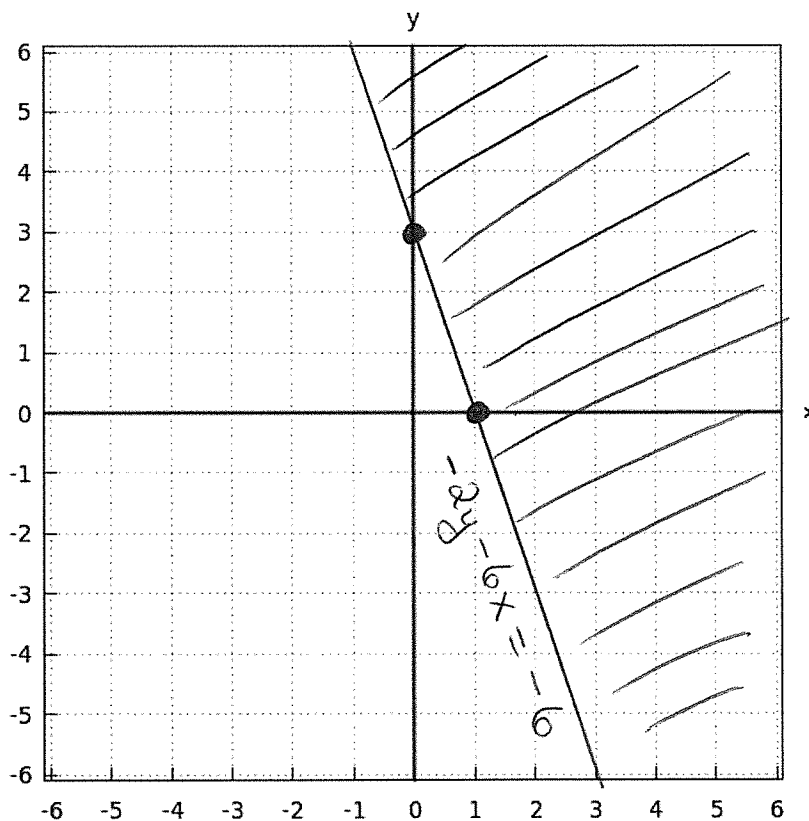
and

$$0 > -6.$$

(b) Graph the feasible set for the inequality on the following grid: [4 pts]

Look at
 $-2y - 6x = -6$

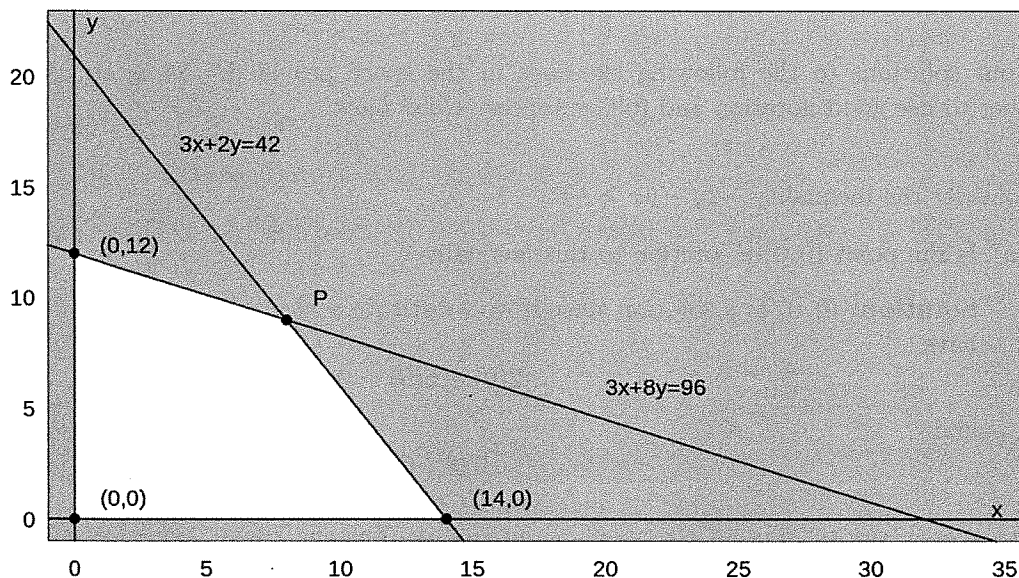
x	y
0	3
1	0



2. A friend asks for your help in maximizing the objective function $f = 20x + 25y$ subject to the constraints

$$3x + 8y \leq 96, \quad 3x + 2y \leq 42, \quad x \geq 0, \quad y \geq 0.$$

Below is the graph of the feasible set for the constraint system:



- (a) What are the 4 vertices (corners) of the feasible set? [5 pts]

Solution: 3 of the 4 vertices can be read off from the provided graph: $(0,0)$, $(0,12)$ and $(14,0)$. The remaining vertex is the point of intersection of the lines $3x + 8y = 96$ and $3x + 2y = 42$. We will find the point of intersection via Elimination. Subtracting the first equation from the second yields

$$6y = 54 \implies y = 9.$$

Substituting $y = 9$ into the second equation then gives

$$3x + 2(9) = 42 \implies 3x + 18 = 42 \implies 3x = 24 \implies x = 8.$$

Thus, the 4th vertex is the point $(8,9)$.

- (b) Find the maximum value of f and the x and y values that yield the maximum. [3 pts]

Solution: By The Fundamental Theorem of Linear Programming, the maximum value (if it exists) must occur at a vertex of the feasible set. We have

Corner	x	y	$f = 20x + 25y$
$(0,0)$	0	0	$20(0) + 25(0) = 0$
$(14,0)$	14	0	$20(14) + 25(0) = 280$
$(0,12)$	0	12	$20(0) + 25(12) = 300$
$(8,9)$	8	9	$20(8) + 25(9) = 385$

Thus, the maximum value is 385 which occurs when $x = 8$ and $y = 9$.

3. Two printing companies offer the following prices for a large printing run: Company A charges a setup fee of \$5 and 3 cents for each page printed; Company B charges no setup fee but charges 4 cents per page printed.

Let n denote the number of pages printed, P_A denote the price charged in pennies by Company A, and P_B denote the price charged in pennies by Company B.

- (a) Give a linear equation that expresses the relationship between n and P_A . [1 pt]

Solution: Since we are working in cents, note that the setup fee is $5 \times 100 = 500$ cents. Thus, $P_A = 500 + 3n$.

- (b) Give a linear equation that expresses the relationship between n and P_B . [1 pt]

Solution: $P_B = 4n$

- (c) Which company gives the cheaper price to print 1000 pages? [3 pts]

Solution: For $n = 1000$, we have

$$P_A = 500 + 3(1000) = 3500$$

$$P_B = 4(1000) = 4000$$

Therefore, Company A gives the cheaper price.

- (d) What is the number of pages for which each company charges the same price? [2 pts]

Solution: Both companies charge the same price when $P_A = P_B$. That is, when

$$500 + 3n = 4n \iff n = 500.$$

