MATH 7470: Rings And Modules
Course Information Sheet and Syllabus
Fall 2019

"If one proves the equality of two numbers a and b by showing first that $a \leq b$ and then that $a \geq b$, it is unfair; one should instead show that they are really equal by disclosing the inner ground for their equality." — Emmy Noether

BASIC COURSE DETAILS

Instructor: Dr. Susan Cooper

Class Times and Location: Tuesdays & Thursdays 1:00 – 2:15 p.m., 305 Tier

Credit Hours: 3

Pre-Requisites: Permission of department.

Note: MATH 7470 may not be held with MATH 4470.

INSTRUCTOR CONTACT INFORMATION

Instructor: Dr. Susan Cooper **Email:** susan.cooper@umanitoba.ca

Office: Machray Hall, Room 476 **Office Phone:** (204) 474–9701

Office Hours: Tuesdays 3:30 – 4:30 p.m. & Wednesdays 9:30 – 10:30 a.m.; or by appointment

Correspondences and Appointments: The most reliable way to contact me is via email. I will reply to an email within 24 hours of receiving it Monday – Thursday; an email received Friday – Sunday will receive a reply the following Monday. *All appointments are to be made via email.*

COURSE DESCRIPTION AND GOALS

"The value of a principle is the number of things it will explain." — Ralph Waldo Emerson

University of Manitoba Course Calendar Description: The general theory of (non-commutative) rings, modules and algebras.

General Course Description and Goals: One of the great advantages of studying mathematics is that it helps one develop the ability to handle abstract ideas. Abstract algebra allows us to cultivate this ability with concrete examples, mathematical rigor, and beautiful applications. The study of algebra involves topics that are important branches of mathematics that are thriving and intriguing in their own right. Indeed, algebra hosts many applications which intersect subjects such as number theory, geometry, and analysis. For example, central in ring theory is an ideal which was introduced by Kummer in his work on the famous Fermat's Last Theorem. A second example comes from 19th century work on solving polynomial equations - work that led to Galois theory.

MATH 7470 is a one-term course that provides an in-depth study of the theory of rings and modules. For each topic we study sets that are endowed with algebraic structures. For rings, there are two operations (addition and multiplication) with an accompanying distributive law. The most familiar

examples of rings include the integers and polynomial rings. Modules are Abelian groups which are acted upon by rings. As such, a module over a ring is a generalization of a vector space over a field. In many ways, the structure of a ring is encoded in the properties of its modules. A deep understanding of these topics is fundamental to higher-level mathematics.

By considering concrete examples, you will make conjectures and then try to verify or disprove them. You will gain facility and become confident that you can *do* mathematics and you will experience the joy of discovering hidden patterns and mathematical truths. After successful completion of the course, students will be able to state, prove, apply fundamental theorems, and construct and work with a variety of concrete examples. In addition, students will be well-prepared for follow-up work in algebra.

COURSE MATERIALS

Recommended Textbook: *Abstract Algebra*, 3rd edition, by David S. Dummit and Richard M. Foote

Note: At times, we may supplement the course with material not presented in the textbook. Thus the lecture notes are the main resource for this course.

Course Web-Page: We will use UM Learn and the instructor's web-page which can be found at
`http://server.math.umanitoba.ca/~coopers5/courses_umanitoba/math4470_7470_f19.html`

ASSESSMENTS AND COURSE GRADES

Problem Sets and Readings: Mathematics is not a spectator sport. The best way to learn mathematics is by doing mathematics! To encourage constant engagement with the course material, a collection of exercises and readings will be assigned and collected on a weekly basis. Typically, a Problem Set will be due by 9:00 a.m. on the Wednesday of the following week the assignment is distributed. You may submit your solutions either in-class or to the Department of Mathematics (*with date and time of submission noted*). As this is a cross-listed course with MATH 4470, students registered in MATH 7470 will be required to complete more problems than those registered in MATH 4470. A subset of the solutions will be graded based on correctness, completeness, and quality of exposition (clarity, style/creativity, conciseness, etc.). Any necessary special grading rules will be provided beforehand. Moreover, students will receive ample feedback before the Voluntary Withdrawal Deadline.

It is highly recommended that you carefully read the material and complete all the exercises in a Problem Set well before the deadline in order to keep on track and make sure that you have solid foundations before moving on to the next topic. The Problem Sets are intended to gauge your understanding of the material while presenting opportunities for you to practice the fine art of communicating mathematics. Please note that all feedback is meant to *improve* your mathematical abilities and communication.

In many areas of life, we deepen our understanding via discussions with others and a variety of resources. Although you are encouraged to work together on Problem Sets and to discuss ideas with the instructor, you are expected to submit solutions that are written individually and in your own words: see the section entitled "Academic Integrity" and the handout entitled "Guidelines for Problem Sets" for further expectations and more information.

Participation Presentations: The depth to which one truly understands a topic is reflected in how well one can teach the topic to another. Each student will present agreed-upon material in one 75-minute lecture. Students are to submit lecture notes to the instructor for approval at least 48 hours in advance of the presentation for approval. A final copy of the presentation notes is to be submitted by 9:00 a.m. the day following the presentation and will be shared with classmates on UM Learn. The lecture notes must provide a list of all resources that were used in preparation of the presentation. Students are required to typeset their notes using LaTeX. In consultation with the instructor, the date for each student's presentation must be agreed upon no later than October 1, 2019. A detailed rubric for the presentations and the final copy of the lecture notes will be distributed before the first student presentation. At the very least, presentations and notes will be judged for organization, mathematical content, and delivery. Written feedback will be provided and is meant to *improve* presentation skills along with the correctness of the mathematics presented.

Presentation Peer-Reviews: In order to encourage engagement and to give well-rounded feedback on the student presentations, anonymous feedback from the entire class will be given on the presentations. Students will receive full credit for completing a presentation feedback form with constructive comments for all of the presentations.

Class Attendance and Participation: Your understanding of the course material will be greatly supported by regular attendance and engagement in class meetings. Although you are expected to attend every class and to fully participate in class discussions, attendance will not be taken or be used in the calculation of course grades. However, you are responsible for any missed material when absent. In addition, it is often not easy for students to give presentations and so your support and encouragement via attendance is appreciated. Indeed, you wouldn't want to put a lot of time and energy into preparing for a lecture you are to give and then have no audience - please show respect for your classmates and attend class and arrive on time.

Evaluation Scheme and Letter Grades: Final course grades will be determined as follows:

Task	**Percentage of Grade**
Problem Sets	70%
Presentations	10%
Presentation Lecture Notes	10%
Presentation Peer-Reviews	10%

Below are the minimum cut-off ranges for grades; these may decrease at the instructor's discretion.

Letter Grade	**Percentage Out Of 100**	**Final Grade Point Value**
A+	95–100	4.5
A	85–94	4.0
B+	80–84	3.5
B	72–79	3.0
C+	65–71	2.5
C	60–64	2.0
D	50–59	1.0
F	Less than 50	0

Deferred Work: Late acceptance of work and rescheduling of presentations will only be granted for unavoidable, documented circumstances as described below:

Circumstance	Required documentation
Illness or other medical situation	Official note from clinic, hospital, doctor, nurse, or other health care provider
Military service	Official military activation orders
Funeral or other family emergency	Official documentation from newspaper, funeral, or medical official
Sports or other official U of M activity	Official documentation from U of M athletics or activity's faculty adviser

Students who wish to request a late acceptance or presentation to be rescheduled must contact the instructor within 24 hours of the due date/time (initial email contact is sufficient). Please note that recreational activities do not qualify for deferred work. If you have a pre-existing conflict with an assessment, you are expected to make alternative arrangements *beforehand*.

COURSE SCHEDULE AND IMPORTANT DATES

The topics schedule below is subject to change at the discretion of the instructor and/or based on learning needs of the students (subject to Section 2.8 of Responsibilities Of Academic Staff With Regard To Students). Assessment dates will not change.

Dates	**Topic**	**Problem Set Due**
Sept. 4–6	Module Theory definitions, examples, quotients, homomorphisms	
Sept. 9–13	Module Theory quotients, homomorphisms, generation, direct sums, free	# 1 Sept. 11
Sept. 16–20	Module Theory generation, direct sums, free, tensor products	# 2 Sept. 18
Sept. 23–27	Module Theory tensor product, exact sequences (projective, injective, flat)	# 3 Sept. 25
Sept. 30–Oct. 4	Module Theory exact sequences (projective, injective, flat), over PIDs	# 4 Oct. 2
Oct. 7–11	Module Theory modules over PIDs	# 5 Oct. 9
No Classes: Monday, October 14 (Thanksgiving Day)		
Oct. 14–18	Module Theory modules over PIDs	# 6 Oct. 16
Oct. 21–25	Module Theory & Commutative Rings modules over PIDs, Noetherian rings, localization	# 7 Oct. 23
Oct. 28–Nov. 1	Commutative Rings localization, Artinian rings, discrete valuation rings	# 8 Oct. 30
Nov. 4–8	Commutative Rings discrete valuation rings, dedekind domains	# 9 Nov. 6
No Classes: November 11–15 (Remembrance Day & Fall Term Break)		
Voluntary Withdrawal Deadline: Monday, November 18		
Nov. 18–22	Commutative Rings & Homological Algebra dedekind domains, Ext and Tor	# 10 Nov. 20
Nov. 25–29	Homological Algebra & Category Theory Ext and Tor, cohomology of groups, definitions, examples	# 11 Nov. 27
Dec. 2–6	Category Theory definitions, examples	# 12 Dec. 4

UNIVERSITY AND DEPARTMENT OF MATHEMATICS SUPPORT OFFICES AND POLICIES

A list (entitled Schedule "A") of supports available to students, including mathematical support, can be found on the course web-page(s).

EXPECTATIONS

Recording Class Lectures: Susan Cooper and the University of Manitoba hold copyright over the course materials, presentations, and lectures which form part of this course. No audio or video recording of lectures or presentations is allowed in any format (including photographs), openly or surreptitiously, in whole or in part without permission. Course materials (both paper and digital) are for the participant's private study and research.

Using Copyrighted Material: Please respect copyright. We will use copyrighted material in this course. I have ensured that the content I use is appropriately acknowledged and is copied in accordance with copyright laws and University guidelines. Copyrighted works, including those created by me, are made available for private study and research and must not be distributed in any format without permission. Do not upload copyrighted works to a learning management system (such as UM Learn), or any website, unless an exception to the *Copyright Act* applies or written permission has been confirmed. For more information, see the University's Copyright Office web-site at `http://umanitoba.ca/copyright/` or contact `um_copyright@umanitoba.ca`.

Course Technology: It is the University of Manitoba policy that all technology resources are to be used in a responsible, efficient, ethical and legal manner. During class meetings and for Problem Sets and presentations, it is expected that you only use technology for educational purposes and that the only technology used is approved by myself and/or the University of Manitoba Student Accessibility Services. You should not participate in personal direct electronic message/posting activities (such as e-mail, texting, video, social networking, etc.) during scheduled class time – this is not only in your best interests for understanding the course material but is respectful behaviour for your classmates. If you absolutely need to take an expected call, then please use the vibrate mode on your cell phone and leave the classroom before using the phone.

Class Communication: You are required to obtain and use your University of Manitoba email account for all communication between yourself and the university. All communication must comply with the Electronic Communication with Students Policy.

Student Accessibility Services: The University of Manitoba is committed to providing an accessible academic community. *Students Accessibility Services (SAS)* offers academic accommodation supports and services such as note-taking, interpreting, assistive technology and exam accommodations. Students who have, or think they may have, a disability (e.g. mental illness, learning, medical, hearing, injury-related, visual) are invited to contact SAS to arrange a confidential consultation. Students are welcome to meet with the instructors to discuss the accommodations recommended by SAS.

Student Accessibility Services
`http://umanitoba.ca/student-supports/accessibility`
520 University Centre
Phone: (204) 474–7423
Email: Student_accessibility@umantioba.ca

Classroom Atmosphere: A part of learning is making mistakes. We want to establish a classroom atmosphere where the inevitable false starts and mistakes become an opportunity to improve – not an opportunity for embarrassment. Please be constructive and polite in questioning your colleagues.

Academic Integrity: You are expected to be academically honest. This means, for example, providing a list of the people (if any) with whom you worked and providing a list of sources other than the textbook (if any) that you used to complete an assignment. Although you are encouraged to work together, you should never submit anything that you do not understand or is not written in your own words. The following excerpt about Academic Honesty is taken from the Department of Mathematics web-page:

The Department of Mathematics, the Faculty of Science and the University of Manitoba regard acts of academic dishonesty in quizzes, tests, examinations or assignments as serious offences and may assess a variety of penalties depending on the nature of the offence. Acts of academic dishonesty include bringing unauthorized materials into a test or exam, copying from another student, plagiarism and examination personation. Students are advised to read the sections entitled "Academic Integrity" and "Final Examinations" (in particular, the section "Examination Personations") in the "General Academic Regulations" section of the current Undergraduate Calendar.

Penalties for violation include being assigned a grade of zero on a test or assignment, being assigned a grade of "F" in a course, compulsory withdrawal from a course or program, suspension from a course/program/faculty or even expulsion from the University. For specific details about the nature of penalties that may be assessed upon conviction of an act of academic dishonesty, students are referred to University Policy 1202 (*Student Discipline Bylaw*) and to the Department of Mathematics policy concerning minimum penalties for acts of academic dishonesty. Information regarding Academic Integrity is also available from the Faculty of Science web-page. Moreover, students have access to information via the course GRAD 7500 available through the Faculty of Graduate Studies.

Students are encouraged to visit the University of Manitoba Academic Integrity site for further information.

Other Expectations and Tips for Success: I ask that you have a well-defined sense of professionalism, that you always put forth your best effort, and that you develop a sense of responsibility to your educational community. I ask that you exhibit a persistent desire to learn. In return I will provide you with significant support. Also:

- Be positive, open, and responsive to feedback.
- Be an active participant - mathematics is learned by doing; this includes participating fully in classroom activities (please, turn your cell phones off during class), completing the Problem Sets, participating via Problem Set Presentations, critically thinking about the mathematics during and outside of class. *In order for this class to be successful, it is imperative that you commit to attending class/tutorial sessions regularly, that you commit to preparing beforehand for class meetings, and that you commit to participating in class meetings!*
- Be/become a "risk taker".
- Be committed.
- Be patient with yourself - it takes time to master newly learned things. Ask for assistance when it is needed. Constantly try to improve yourself as a mathematician.
- Starting with the first class, study in-depth and regularly.
- It is tempting to just copy available solutions. However, struggling through the exercises on your own is an important phase of the learning process.
- Get help as soon as you need it: ask questions in class and office hours; form a study group with your classmates; read alternate resources.
- Like in all areas of life, constructive feedback can be difficult to digest and accept. Please know that the feedback provided in this course is meant to *improve* your mathematical solutions and communication. Please take the feedback seriously and apply it to your future work.
- Everyone wants you to succeed. Please speak with me regarding any concerns you may have.
- Relax and have fun with the course!