

Problem Set 9

Due: 9:00 a.m. on Wednesday, November 6

Instructions: MATH 7470 students should submit solutions to all of the following problems and MATH 4470 students should submit solutions to only those marked with a “U”. A subset of the problems will be graded. Be sure to adhere to the expectations outlined on the sheet *Guidelines for Problem Sets*. You may submit your solutions either in-class or to the Department of Mathematics (*with date and time of submission noted*).

Exercises: For this Problem Set, let R be a commutative ring with identity and let M be a unital R -module.

1U. Let R be an integral domain and let M be a projective R -module.

(a) Prove that M is torsion-free (that is, $\text{Tor}(M) = \{0_M\}$).

(b) Assume further that R is a P.I.D. and that M is finitely generated. Prove that M is a free R -module.

2. Assume that R is a Principal Ideal Domain and that M is a finitely generated R -module. Prove that if M is flat over R , then M is a free R -module.