Problem Set 7

Due: 9:00 a.m. on Wednesday, October 23

Instructions: MATH 7470 students should submit solutions to all of the following problems and MATH 4470 students should submit solutions to only those marked with a "U". A subset of the problems will be graded. Be sure to adhere to the expectations outlined on the sheet *Guidelines for Problem Sets*. You may submit your solutions either in-class or to the Department of Mathematics (*with date and time of submission noted*).

Exercises: For this Problem Set, let R be a commutative ring with identity.

1U. Let M be an R-module. Prove that the following conditions are equivalent:

(i) M is flat over R.

(ii) For every injective R-module homomorphism $g' : N' \longrightarrow N$, the induced homomorphism $id_M \otimes g' : M \otimes_R N' \longrightarrow M \otimes_R N$ is injective.

(iii) For every short exact sequence

$$0 \longrightarrow N' \xrightarrow{g'} N \xrightarrow{g} N'' \longrightarrow 0$$

of R-module homomorphisms, the induced sequence

$$0 \longrightarrow M \otimes_R N' \xrightarrow{id_M \otimes g'} M \otimes_R N \xrightarrow{id_M \otimes g} M \otimes_R N'' \longrightarrow 0$$

is exact.

2. Let M be an R-module and $S \subseteq R$ be a multiplicatively closed subset of R. Recall that in class we showed that every element of $S^{-1}R \otimes_R M$ is of the form $\left(\frac{1}{s}\right) \otimes_R m$ for some $s \in S$ and $m \in M$. Prove that there exists a unique R-module isomorphism $f : S^{-1}R \otimes_R M \to S^{-1}M$ with $f(\frac{r}{s} \otimes_R m) = \frac{rm}{s}$ for all $r \in R, m \in M$ and $s \in S$.