Problem Set 6

Due: 9:00 a.m. on Wednesday, October 16

Instructions: MATH 7470 students should submit solutions to all of the following problems and MATH 4470 students should submit solutions to only those marked with a “U”. A subset of the problems will be graded. Be sure to adhere to the expectations outlined on the sheet *Guidelines for Problem Sets*. You may submit your solutions either in-class or to the Department of Mathematics (*with date and time of submission noted*).

Exercises: For this Problem Set, let R be a ring with identity.

1U. Let L, M, and N be unitary R-modules. Let $f : M \to N$ be an R-module isomorphism. Prove that the map $f^* : \mathrm{Hom}_R(N, L) \to \mathrm{Hom}_R(M, L)$ is an isomorphism.

2U. (Dummit and Foote §10.5 #3) Let P_1 and P_2 be R-modules. Prove that $P_1 \oplus P_2$ is a projective R-module if and only if both P_1 and P_2 are projective. You may assume the fact that any direct sum of free R-modules is free.

3U. (Dummit and Foote §10.5 #6) Prove that the following are equivalent:

(i) Every R-module is projective.

(ii) Every R-module is injective.