Problem Set 3
Due: 9:00 a.m. on Wednesday, September 25

Instructions: MATH 7470 students should submit solutions to all of the following problems and MATH 4470 students should submit solutions to only those marked with a "U". A subset of the problems will be graded. Be sure to adhere to the expectations outlined on the sheet *Guidelines for Problem Sets*. You may submit your solutions either in-class or to the Department of Mathematics (*with date and time of submission noted*).

Exercises: From the textbook *Abstract Algebra*, 3rd edition, by David S. Dummit and Richard M. Foote.

Unless otherwise instructed, throughout assume that R is a ring with identity $1 \neq 0$ and that M is a left R-module.

1U. (Dummit and Foote §10.3 #7) Let N be a submodule of M. Prove that if both M/N and N are finitely generated then so is M.

2U. (Dummit and Foote §10.3 #13) Let R be a commutative ring and let F be a free R-module of finite rank. Prove the following isomorphism of R-modules: $\mathrm{Hom}_R(F, R) \cong F$.

3. A non-zero unitary R-module M is called *simple* if its only submodules are 0 and M.

   (iU.) Prove that if M is a simple R-module, then M is cyclic.

   (iiU.) Let $\alpha : M \to N$ be a homomorphism between simple R-modules. Prove that α is either 0 or an isomorphism.

   (iii) Assume that R is commutative. Prove that M is a simple R-module if and only if there is a maximal ideal $I \subseteq R$ such that $M \cong R/I$ (as R-modules).