Problem Set 2

Due: 9:00 a.m. on Wednesday, September 18

Instructions: MATH 7470 students should submit solutions to all of the following problems and MATH 4470 students should submit solutions to only those marked with a "U". A subset of the problems will be graded. Be sure to adhere to the expectations outlined on the sheet *Guidelines for Problem Sets*. You may submit your solutions either in-class or to the Department of Mathematics (*with date and time of submission noted*).

Exercises: From the textbook *Abstract Algebra*, 3rd edition, by David S. Dummit and Richard M. Foote.

Unless otherwise instructed, throughout assume that R is a ring with identity $1 \neq 0$ and that M is a left R-module.

1U. (Dummit and Foote §10.2 #1) Use the submodule criterion to show that kernels and images of R-module homomorphisms are submodules.

2U. (Dummit and Foote §10.2 #9) Let R be a commutative ring. Prove that $\mathrm{Hom}_R(R, M)$ and M are isomorphic as left R-modules. [Show that each element of $\mathrm{Hom}_R(R, M)$ is determined by its value on the identity of R.]

3U. (Dummit and Foote §10.2 #13) An ideal J is called *nilpotent* if J^n is the zero ideal for some $n \geq 1$. Let I be a nilpotent ideal in a commutative ring R, let M and N be R-modules and let $\varphi : M \to N$ be an R-module homomorphism. Show that if the induced map $\overline{\varphi} : M/IM \to N/IN$ is surjective, then φ is surjective.

4. (Dummit and Foote §10.2 #3) Give an explicit example of a map from one R-module to another which is a group homomorphism but not an R-module homomorphism.