Problem Set 12
Due: 9:00 a.m. on Wednesday, December 4

Instructions: MATH 7470 students should submit solutions to all of the following problems and MATH 4470 students should submit solutions to only those marked with a "U". A subset of the problems will be graded. Be sure to adhere to the expectations outlined on the sheet *Guidelines for Problem Sets*. You may submit your solutions either in-class or to the Department of Mathematics (*with date and time of submission noted*).

Exercises: For this Problem Set, assume that all rings are non-zero, commutative, and contain an identity $\neq 0$.

1U. (Dummit and Foote §16.2 #1) Suppose R is a Discrete Valuation Ring with respect to the valuation ν on the fraction field K of R. If $x, y \in K$ with $\nu(x) < \nu(y)$ prove that $\nu(x + y) = \min(\nu(x), \nu(y))$.

2. (Dummit and Foote §16.2 #2) Suppose R is a Discrete Valuation Ring with unique maximal ideal M and quotient $F = R/M$. For any $n \geq 0$ show that M^n/M^{n+1} is a vector space over F and that $\dim_F(M^n/M^{n+1}) = 1$.