Problem Set 10

Due: 9:00 a.m. on Wednesday, November 20

Instructions: MATH 7470 students should submit solutions to all of the following problems and MATH 4470 students should submit solutions to only those marked with a "U". A subset of the problems will be graded. Be sure to adhere to the expectations outlined on the sheet *Guidelines for Problem Sets*. You may submit your solutions either in-class or to the Department of Mathematics (*with date and time of submission noted*).

Exercises: For this Problem Set, assume that all rings are non-zero, commutative, and contain an identity $\neq 0$.

1U. (Dummit and Foote §7.4 #11) Let I and J be ideals of R and assume P is a prime ideal of R that contains IJ. Prove that either I or J is contained in P.

2U. (Dummit and Foote §7.4 #26) In class we defined the nilradical of R as the intersection of all prime ideals of R. This exercise demonstrates an equivalent definition: Recall that an element $x \in R$ is *nilpotent* if $x^n = 0$ for some $n \in \mathbb{Z}^+$. You may assume the fact that the set of nilpotent elements form an ideal – called the *nilradical* of R. Prove that a prime ideal in R contains every nilpotent element. Deduce that the nilradical of R is contained in the intersection of all the prime ideals of R.

3. (Dummit and Foote §16.1 #4) Prove that an Artinian integral domain is a field.