Problem Set 1
Due: 9:00 a.m. on Wednesday, September 11

Instructions: MATH 7470 students should submit solutions to all of the following problems and MATH 4470 students should submit solutions to only those marked with a "U". A subset of the problems will be graded. Be sure to adhere to the expectations outlined on the sheet *Guidelines for Problem Sets*. You may submit your solutions either in-class or to the Department of Mathematics (*with date and time of submission noted*).

Exercises: From the textbook *Abstract Algebra*, 3rd edition, by David S. Dummit and Richard M. Foote.

Unless otherwise instructed, throughout assume that R is a ring with identity 1 and that M is a left R-module.

1U. (Dummit and Foote §10.1 #5) For any left ideal I of R define

$$IM = \left\{ \sum_{\text{finite}} a_i m_i \mid a_i \in I, m_i \in M \right\}$$

to be the collection of all finite sums of elements of the form am where $a \in I$ and $m \in M$. Prove that IM is a submodule of M.

2U. (Dummit and Foote §10.1 #8) An element m of the R-module M is called a *torsion element* if $rm = 0$ for some nonzero element $r \in R$. The set of torsion elements is denoted

$$\text{Tor}(M) = \{m \in M \mid rm = 0 \text{ for some nonzero } r \in R\}.$$

(a) Prove that if R is an integral domain then $\text{Tor}(M)$ is a submodule of M (called the *torsion* submodule of M).

(b) Give an example of a ring R and an R-module M such that $\text{Tor}(M)$ is not a submodule. [Consider the torsion elements in the R-module R.]

(c) If R has zero divisors show that every nonzero R-module has nonzero torsion elements.

3. (Dummit and Foote §10.1 #9) If N is a submodule of M, the *annihilator of N in R* is defined to be

$$\{r \in R \mid rn = 0 \text{ for all } n \in N\}.$$

Prove that the annihilator of N in R is a 2-sided ideal of R.