Problem Set 9

Due: 10:00 a.m. on Thursday, November 21

Instructions: Submit solutions to all of the following exercises. A subset of the problems will be graded. Be sure to adhere to the expectations outlined on the sheet *Guidelines for Problem Sets*. You may submit your solutions either in-class or to the Department of Mathematics (*with date and time of submission noted*).

Exercises: Be sure to show all of your work and fully justify your answers and reasoning.

1. There are 12 girls and 9 boys at a party. For a certain game, one must line up 5 of the girls and 3 of the boys in a row, left-to-right, in such a way that no two boys are side-by-side. In how many ways can this be done?

2. Suppose there is an 8-card poker game created using the standard 52 deck of cards. We want to determine which of the following are less likely (and therefore should be higher ranked). Do this by determining how many of the following hands are possible.

   (a) A double 4 of a kind (two different ranks of cards, each with four of a kind).

   (b) A super full house (two ranks with three of a kind and one rank with a pair).

   (c) A super straight (eight straight ranks such as $3-4-5-6-7-8-9-10$ or $Ace-2-3-4-5-6-7-8$ or $6-7-8-9-10-Jack-Queen-King-Ace$).

   (d) A super flush (eight cards of all the same suit).

   For parts (c) and (d) do not worry about whether the hand will satisfy a higher ranking hand - just find the number of ways you can satisfy the given property.

3. What is the coefficient of x^5y^2 in the expansion of each of the following expressions:

   (a) $(2x-3y)^7$

   (b) $(1+x+y)^{10}$

4. Suppose you are playing a game of SCRABBLE.

   (a) If you have the tiles A, A, A, B, B, C, D, then how many 7-letter "words" can be formed with these letters?

   (b) How many 7-letter "words" can be formed if you have a blank tile instead of the D from part (a)? A blank tile can be used as any letter.

5. From a group of n people, two non-empty committees (call them A and B) are to be formed, possibly leaving some members of the group on neither committee.

   (a) In how many ways can this be done if it is permitted that the two committees can overlap?

   (b) In how many ways can this be done if the two committees must be disjoint?

6. Give an algebraic and a combintorial proof for the identity:

$$P(n,k) = P(n-1,k) + kP(n-1,k-1).$$