Problem Set 7
Due: 10:00 a.m. on Thursday, October 31

Instructions: Submit solutions to all of the following exercises. A subset of the problems will be graded. Be sure to adhere to the expectations outlined on the sheet *Guidelines for Problem Sets*. You may submit your solutions either in-class or to the Department of Mathematics (*with date and time of submission noted*).

Exercises: Be sure to show all of your work and fully justify your answers and reasoning.

1. Consider the digraph $G(V, E)$ given by

$$V = \{a, b, c, d, e, f\} \quad \text{and} \quad E = \{(a, b), (a, d), (d, e), (b, e), (d, f), (e, f), (b, c), (c, a)\}.$$

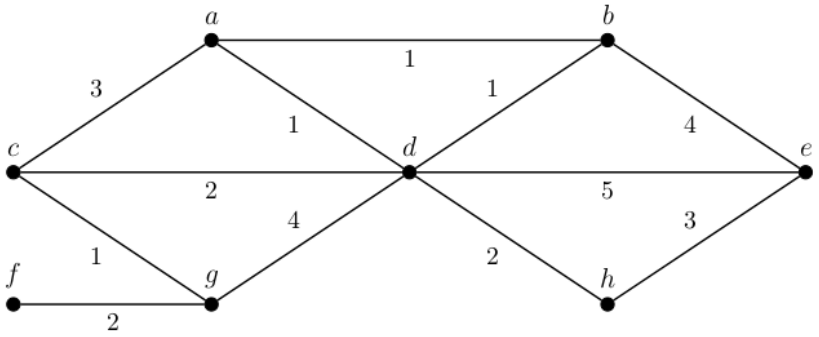
   Use WARSHALL's algorithm to determine the path matrix for $G(V, E)$. Show the matrices P_k at each step.

2. Consider the following edge-weight matrix for a simple graph G:

$$\begin{bmatrix} 0 & 7 & 14 & 9 & \infty & \infty & \infty \\ 7 & 0 & \infty & 10 & 15 & \infty & \infty \\ 14 & \infty & 0 & 2 & \infty & 3 & 9 \\ 9 & 10 & 2 & 0 & 11 & 6 & \infty \\ \infty & 15 & \infty & 11 & 0 & 5 & 6 \\ \infty & \infty & 3 & 6 & 5 & 0 & 4 \\ \infty & \infty & 9 & \infty & 6 & 4 & 0 \end{bmatrix}.$$

   Perform DIJKSTRA's algorithm with starting node g and final node a. Record the steps in a table as demonstrated in class and the course notes. State $d(a)$ as a final conclusion.

3. Let G be the following weighted graph:



   Find a minimal spanning tree of G by:

   (a) using PRIM's algorithm starting at vertex a;

   (b) using KRUSKAL's algorithm.

   For both parts (a) and (b), at each step if there is a choice between two equal edges, choose the one earlier in the alphabet. Also, your output should be the graphs in each step as demonstrated in class.