

Problem Set 4

Due: 10:00 a.m. on Thursday, October 3

Instructions: Submit solutions to all of the following exercises. A subset of the problems will be graded. Be sure to adhere to the expectations outlined on the sheet *Guidelines for Problem Sets*. You may submit your solutions either in-class or to the Department of Mathematics (*with date and time of submission noted*).

Exercises: Be sure to show all of your work and fully justify your answers and reasoning.

1. Use truth tables to determine whether the following arguments are valid:

(a)

$$\begin{array}{l}
 B \vee C \\
 B \implies A \\
 C \implies A \\
 \hline
 A
 \end{array}$$

(b)

$$\begin{array}{l}
 A \implies B \\
 \neg A \implies C \\
 \neg B \\
 \hline
 C
 \end{array}$$

(c)

$$\begin{array}{l}
 (A \vee C) \implies (B \vee C) \\
 \hline
 A \implies B
 \end{array}$$

2. Determine whether the following arguments are valid:

(a)

All dogs are fish.
My table is a dog.

My table is a fish.

(b)

If I insult my sister, my brother will be upset.
I did not insult my sister.

My brother is not upset.

3. Fun with WFFs!

(a) Given the truth table below, find a well-formed formula (WFF) for W :

A	B	C	W
T	T	T	F
T	T	F	F
T	F	T	F
T	F	F	T
F	T	T	T
F	T	F	T
F	F	T	F
F	F	F	F

(b) Provide WFFs which are logically equivalent to:

- (i) $A \not\Rightarrow B$
- (ii) \perp
- (iii) \top
- (iv) $A \mid B$
- (v) $A \downarrow B$

(c) It turns out that we only need the symbols \neg and \wedge to write all WFFs. Verify this by creating logical equivalent formulas to

- (i) $A \implies B$
- (ii) $A \implies ((A \wedge B) \vee C)$

using only atoms, \neg and \wedge .

4. Prove the following theorems from Boolean Algebra:

- (a) For all expressions a , we have $a \cdot 0 = 0$.
- (b) $0 + 0 = 0$.