

Problem Set 2

Due: 10:00 a.m. on Thursday, September 19

Instructions: Submit solutions to all of the following exercises. A subset of the problems will be graded. Be sure to adhere to the expectations outlined on the sheet *Guidelines for Problem Sets*. You may submit your solutions either in-class or to the Department of Mathematics (*with date and time of submission noted*).

Exercises: Be sure to show all of your work and fully justify your answers and reasoning.

1. Use the floor function to determine the number of multiples of 11 which are less than or equal to 1000.
2. Given a positive integer m , let R be the relation on \mathbb{Z} defined by

$$R = \{(a, b) \in \mathbb{Z}^2 \mid a \equiv b \pmod{m}\}.$$

- (a) Show that R is an equivalence relation.
 - (b) If $m = 10$, determine $A_3 \cap \{1, 2, \dots, 100\}$ where A_3 is the equivalence class of 3.
3. Let m be a positive integer and a, b, c be integers. Assume that $a \equiv b \pmod{m}$. Prove that $a + c \equiv b + c \pmod{m}$.
 4. For the following, use the Euclidean Algorithm as demonstrated in class.
 - (a) Find the greatest common divisor of $a = 57,970$ and $b = 10,353$.
 - (b) Find integers x and y so that $\gcd(a, b) = ax + by$.
 5. Solve the linear congruence $7x \equiv 3 \pmod{5}$.
 6. Consider the statement “If x is an animal, then x is a mammal”.
 - (a) Define propositional statements A and B such that the above can be written as $A \implies B$.
 - (b) Using A and B , write in symbolic form the converse, contrapositive and the negation of the statement.
 - (c) Write English sentences which state the converse, contrapositive and the negation of the statement.
 - (d) The above statement is false. Give a counter-example. Which of the converse, contrapositive and/or negation is true, if any.
 7. Develop truth table(s) for the propositions $A \wedge (B \vee C)$, $(A \wedge B) \vee (A \wedge C)$ and $(A \wedge B) \vee C$. Use the truth tables to determine which, if any, of the above statements are logically equivalent. (Show all the steps in the truth table, not just the answer.)